Surjunctivity of Algebraic Dynamical Systems

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This is joint work with Siddhartha Bhattacharya and Tullio Ceccherini-Silberstein.

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[BCC-2017] S. Bhattacharya, T. Ceccherini-Silberstein, M. Coornaert, *Surjunctivity and topological rigidity of algebraic dynamical systems*, arXiv:1702.06201

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A dynamical system is a triple (X, Γ, α), where

- X is a compact metrizable topological space,
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- α is a continuous action of Γ on X, i.e., a group morphism $\alpha\colon \Gamma \to \text{Homeo}(X)$.

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To simplify, we write

$$\gamma x := \alpha(\gamma)(x) \quad \forall \gamma \in \Gamma, \forall x \in X,$$

and $(X, \Gamma) := (X, \Gamma, \alpha)$.

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The term surjunctive was created by Gottschalk [Go-1973].

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If (X, Γ) satisfies the descending chain condition, i.e., every decreasing sequence

$$X = X_0 \supset X_1 \supset X_2 \supset \dots$$

of closed invariant subsets eventually stabilizes, then (X, Γ) is surjunctive.

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Minimal d.s. and, more generally, d.s. in which all proper closed invariant subsets are finite satisfy the d.c.c.

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- $S^\Gamma = \{x \colon \Gamma \to S\}$ with the product topology;
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$$(\gamma x)(\gamma') := x(\gamma^{-1}\gamma') \quad \forall \gamma, \gamma' \in \Gamma, \forall x \in S^\Gamma.$$

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It is unknown if the preceding theorem remains valid for any group Γ (Gottschalk conjecture).

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Theorem (CC-2015)

If (X, Γ) is expansive and the periodic points are dense in X, then (X, Γ) is surjunctive.

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Consider the subshift $X \subset \{0,1\}^{\mathbb{Z}}$ consisting of all bi-infinite sequences of 0s and 1s with at most one chain of 1s.
Then $(X, \mathbb{Z})$ is expansive but not surjunctive.

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The map $\tau \colon X \to X$ which replaces each word 10 by 11 is equivariant, continuous, injective but not surjective.

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Example

Let S be any compact metrizable space.
Then periodic points for the $\mathbb{Z}$-shift are dense in $S^{\mathbb{Z}}$.
However, if S is compressible (e.g. S is the unit interval $[0,1]$, or the infinite-dimensional torus $\mathbb{T}^{\mathbb{N}}$, or the Cantor set) then $(X, \mathbb{Z})$ is not surjunctive.

Algebraic dynamical systems

Definition

An algebraic dynamical system is a d.s. (X, Γ), where

- X is a compact metrizable topological group;
- Γ is a countable group acting on X by continuous group morphisms.

Examples of algebraic dynamical systems

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Example (Arnold's cat)

This the a.d.s. $(\mathbb{T}^2, \mathbb{Z})$, where the action of $\mathbb{Z}$ on $\mathbb{T}^2$ is generated by the cat map $(x_1, x_2) \mapsto (x_2, x_1 + x_2)$.

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More generally, if Γ is a countable subgroup of $\mathrm{GL}_n(\mathbb{Z})$, then $(\mathbb{T}^n, \Gamma)$ is an a.d.s.

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Example

More generally, if Γ is a countable subgroup of $\mathrm{GL}_n(\mathbb{Z})$, then $(\mathbb{T}^n, \Gamma)$ is an a.d.s.

Example

Let S be a compact metrizable topological group (e.g. S is a finite discrete group, or S is a compact Lie group, or $S = \mathbb{T}^{\mathbb{N}}$, or $S = \mathbb{Z}_p$ the group of p-adic integers) and Γ a countable ghroup. Then the shift (S^Γ, Γ) is an a.d.s.

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Let M be a countable $\mathbb{Z}[\Gamma]$-module. Let $X_M = \widehat{M}$ denote the Pontryagin dual of M, i.e., the set of all continuous group morphisms

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with the topology of pointwise convergence. Then X_M is a compact metrizable abelian group and Γ acts on X_M by

$$(\gamma x)(m) := x(\gamma^{-1} m) \quad \forall \gamma \in \Gamma, \forall x \in X_M, \forall m \in M.$$

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One says that (X_M, Γ) is the a.d.s. associated with M. This yields a one-to-one correspondence between countable $\mathbb{Z}[\Gamma]$-modules and a.d.s. (X, Γ) with X abelian (cf. [Sch]).

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Remark

When X is abelian and $M = \widehat{x}$, this is equivalent to saying that the $\mathbb{Z}[\Gamma]$-module M is Noetherian.

Topological rigidity

If X is a topological group, one says that a map $f\colon X \to X$ is affine if there exist a continuous group morphism $a\colon X \to X$ an $b \in X$ such that

$$f(x) = a(x) \cdot b \quad \forall x \in X.$$

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Definition

One says that an a.d.s. (X, Γ) is topologically rigid if every equivariant continuous map $f\colon X \to X$ is affine.

Proposition (BCC-2017)

If an a.d.s. is topologically rigid and satisfies the a.d.c.c. then it is surjunctive.

Surjunctivity of algebraic dynamical systems for $\Gamma = \mathbb{Z}^d$

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Theorem (BCC-2017)

Let $(X, \mathbb{Z}^d)$ be an expansive algebraic dynamical system (with X possibly non-abelian). Then $(X, \mathbb{Z}^d)$ is surjunctive.

Surjunctivity of algebraic dynamical systems for $\Gamma = \mathbb{Z}^d$

Theorem (BCC-2017)

Let $(X, \mathbb{Z}^d)$ be an expansive algebraic dynamical system (with X possibly non-abelian). Then $(X, \mathbb{Z}^d)$ is surjunctive.

Proof.

By a result in [KS-1989], periodic points are dense. □

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Theorem (BCC-2017)

Let $(X, \mathbb{Z}^d)$ be an algebraic dynamical system. Suppose that X is abelian and that $(X, \mathbb{Z}^d)$ satisfies the algebraic descending chain condition (i.e., $\widehat{X}$ is Noetherian as a $\mathbb{Z}[\Gamma]$-module). Then $(X, \mathbb{Z}^d)$ is surjunctive.

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Theorem (BCC-2017)

Let (X, Γ) be an algebraic dynamical system. Suppose that X is a solenoid and that (X, Γ) is expansive. Then (X, Γ) is surjunctive.

The ℓ^2-zero divisor conjecture

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If Γ is a countable group, then there is a $\mathbb{C}[\Gamma]$-module structure on

$$\ell^2(\Gamma) := \{f \colon \Gamma \to \mathbb{C} : \sum_{\gamma \in \Gamma} |f(\gamma)|^2 < \infty\}$$

induced by the convolution product $\mathbb{C}[\Gamma] \times \ell^2(\Gamma) \to \ell^2(\Gamma)$.

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One says that a countable group Γ satisfies the ℓ^2-zero-divisor conjecture if $\ell^2(\Gamma)$ is torsion free as a $\mathbb{C}[\Gamma]$-module.

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Every torsion-free elementary amenable group (and hence every torsion-free solvable-by-finite group) satisfies the ℓ^2-zero-divisor conjecture [L-1991].

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Definition

Let (X, Γ) be an a.d.s. and let μ denote the Haar measure on X. One says that (X, Γ) is mixing if

$$\lim_{\gamma \to \infty} \mu(A \cap \gamma B) = \mu(A) \cdot \mu(B)$$

for all measurable subsets $A, B \subset X$.

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Theorem (BCC-2017)

Let (X, Γ) be an algebraic dynamical system such that

- *X is abelian,*
- *(X, Γ) is mixing;*
- *Γ satisfies the ℓ^2-zero-divisor conjecture;*
- *$\widehat{X}$ is a torsion $\mathbb{Z}[\Gamma]$-module.*

Then (X, Γ) is topologically rigid.

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Corollary (BCC-2017)

If in addition (X, Γ) satisfies the a.d.c.c. (i.e., $\widehat{X}$ is a Noetherian $\mathbb{Z}[\Gamma]$-module), then (X, Γ) is surjunctive.

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