

ONLINE ADDENDUM TO THE ARTICLE
“CLASSIFICATION OF REGULAR PARAMETRIZED ONE-RELATION OPERADS”

MURRAY BREMNER AND VLADIMIR DOTSENKO

ABSTRACT. This addendum to [1] collates together the list of polynomial entries of the matrices that are used for the key computations in that paper; these polynomials are not particularly instructive for main proofs, and therefore are separated into an online addendum. We also append the *Magma* script used for an independent verification of our *Maple* computations; that script can be copy-pasted into the online calculator to reproduce the results.

COMPLETE LIST OF THE ENTRIES OF THE REDUCED REPRESENTATION MATRICES

We refer to [1] for notation. All the information in the original cubic relation matrix M is encapsulated in these five very small matrices $B(4), \dots, B(1^4)$ in extremely compressed form.

Entries of $B(4)$.

$$\begin{aligned} B(4)_{11} &= -x_6 x_4 - x_5 x_4 - x_4^2 - x_6 x_3 - x_5 x_3 - 2x_4 x_3 - x_3^2 - x_4 x_2 - x_3 x_2 - x_4 x_1 - x_3 x_1 + x_2 + x_1 \\ B(4)_{12} &= -x_6^3 - 2x_6^2 x_5 - x_6 x_5^2 - 2x_6^2 x_4 - 2x_6 x_5 x_4 - x_6 x_4^2 - 2x_6^2 x_3 - 2x_6 x_5 x_3 - 2x_6 x_4 x_3 - x_6 x_3^2 - 2x_6^2 x_2 - \\ &\quad 2x_6 x_5 x_2 - 2x_6 x_4 x_2 - 2x_6 x_3 x_2 - x_6 x_2^2 - 3x_6^2 x_1 - 4x_6 x_5 x_1 - x_5^2 x_1 - 4x_6 x_4 x_1 - 2x_5 x_4 x_1 - x_4^2 x_1 - 4x_6 x_3 x_1 - \\ &\quad 2x_5 x_3 x_1 - 2x_4 x_3 x_1 - x_3^2 x_1 - 4x_6 x_2 x_1 - 2x_5 x_2 x_1 - 2x_4 x_2 x_1 - 2x_3 x_2 x_1 - x_2^2 x_1 - 3x_6 x_1^2 - 2x_5 x_1^2 - 2x_4 x_1^2 - \\ &\quad 2x_3 x_1^2 - 2x_2 x_1^2 - x_1^3 + x_6 x_3 + x_5 x_3 + x_4 x_3 + x_3^2 - x_6 x_2 - x_5 x_2 - x_4 x_2 - x_2^2 + x_3 x_1 - x_2 x_1 + x_6 + x_4 \\ B(4)_{21} &= -x_6 - x_5 + 1 \\ B(4)_{22} &= -x_6 x_4 - x_5 x_4 - x_4^2 - x_4 x_3 - x_6 x_2 - x_5 x_2 - 2x_4 x_2 - x_3 x_2 - x_2^2 - x_4 x_1 - x_2 x_1 - x_3 - x_1 \end{aligned}$$

Entries of $B(31)$.

$$\begin{aligned} B(31)_{11} &= -x_6^2 x_4 + x_6 x_5 x_4 - x_6 x_4^2 + x_5 x_4^2 - x_6^2 x_3 + x_6 x_5 x_3 + x_6 x_3^2 - x_5 x_3^2 + x_6 x_4 x_1 - x_5 x_4 x_1 + x_6 x_3 x_1 - \\ &\quad x_5 x_3 x_1 - x_6 x_4 - x_5 x_4 + x_4 x_2 + x_4 x_1 + x_2 \\ B(31)_{12} &= x_6^2 x_4 - x_6 x_5 x_4 + x_6 x_4^2 - x_5 x_4^2 + x_6^2 x_3 - x_6 x_5 x_3 - x_6 x_3^2 + x_5 x_3^2 - x_6 x_4 x_1 + x_5 x_4 x_1 - x_6 x_3 x_1 + \\ &\quad x_5 x_3 x_1 - x_6 x_3 - x_5 x_3 + x_3 x_2 + x_3 x_1 + x_1 \\ B(31)_{13} &= -x_6 x_4^2 x_3 - x_4^3 x_3 - x_6 x_4 x_3^2 + x_4 x_3^3 + x_6 x_4 x_3 x_2 + x_4^2 x_3 x_2 - x_3^3 x_2 - x_6 x_4^2 x_1 - x_4^3 x_1 - \\ &\quad x_6 x_4 x_3 x_1 + x_4^2 x_3 x_1 + 2x_4 x_3^2 x_1 + x_6 x_4 x_2 x_1 + x_4^2 x_2 x_1 + x_6 x_3 x_2 x_1 - x_4 x_3 x_2 x_1 - 2x_3^2 x_2 x_1 + x_4^2 x_1^2 + x_4 x_3 x_1^2 - \\ &\quad x_4 x_2 x_1^2 - x_3 x_2 x_1^2 + 2x_6^2 x_5 - x_6^2 x_4 + 2x_6 x_5 x_4 - x_6 x_4^2 - 2x_6 x_5 x_3 + x_6 x_4 x_3 - x_6^2 x_1 - x_6 x_4 x_1 + 2x_5 x_4 x_1 - x_4^2 x_1 + \\ &\quad 2x_6 x_3 x_1 - x_5 x_3 x_1 + x_4 x_3 x_1 - x_3 x_2 x_1 + x_6 x_2^2 - x_5 x_2^2 - x_2 x_1^2 - x_6 x_3 - x_4 x_3 + x_6 x_2 + x_4 x_2 - x_3 x_2 - x_2 x_1 \\ B(31)_{14} &= -x_6 x_4^3 - x_4^4 - x_6 x_4^2 x_3 + x_4^2 x_3^2 + x_6 x_4 x_2^2 + x_4^2 x_2^2 + x_6 x_3 x_2^2 - x_3^2 x_2^2 + x_4^3 x_1 + x_4^2 x_3 x_1 - x_4 x_2^2 x_1 - \\ &\quad x_3 x_2^2 x_1 + x_6^3 + x_6 x_5^2 - 2x_6 x_5 x_4 - x_6^2 x_3 + 2x_6 x_5 x_3 - x_6 x_4 x_3 + x_6 x_2^2 - x_6^2 x_2 - 2x_6 x_5 x_2 + x_6 x_4 x_2 - x_6 x_3 x_2 + \\ &\quad x_6 x_2^2 + x_6^2 x_1 + x_5^2 x_1 + 3x_6 x_4 x_1 - x_5 x_4 x_1 - 2x_6 x_3 x_1 + 2x_5 x_3 x_1 - x_4 x_3 x_1 + x_2^2 x_1 + x_6 x_2 x_1 - x_5 x_2 x_1 - x_3 x_2 x_1 - \\ &\quad x_6 x_1^2 + x_4 x_1^2 - x_3 x_1^2 - x_5 x_3 - x_3^2 + x_5 x_2 - x_4 x_2 + x_3 x_2 - x_2^2 - x_6 - x_4 \\ B(31)_{15} &= -x_6^2 x_4^2 - x_6 x_5 x_4^2 - x_6 x_4^3 - x_5 x_4^3 - x_6^2 x_4 x_3 - x_6 x_5 x_4 x_3 + x_6 x_4 x_3^2 + x_5 x_4 x_3^2 + x_6^2 x_4 x_2 + x_6 x_5 x_4 x_2 + \\ &\quad x_6 x_2^2 x_4 + x_5 x_4^2 x_2 + x_6^2 x_3 x_2 + x_6 x_5 x_3 x_2 - x_6 x_3^2 x_2 - x_5 x_3^2 x_2 + x_6 x_2^2 x_1 + x_5 x_4^2 x_1 + x_6 x_4 x_3 x_1 + x_5 x_4 x_3 x_1 - \\ &\quad x_6 x_4 x_2 x_1 - x_5 x_4 x_2 x_1 - x_6 x_3 x_2 x_1 - x_5 x_3 x_2 x_1 - x_6^3 - 2x_6^2 x_5 - x_6 x_5^2 + x_6 x_4^2 + x_6 x_4 x_3 - x_4^2 x_3 - \\ &\quad 2x_6 x_3^2 + x_3^3 + x_6^2 x_2 + 2x_6 x_5 x_2 - x_6 x_4 x_2 + x_6 x_3 x_2 - x_6 x_2^2 + x_6^2 x_1 + 2x_6 x_5 x_1 + 2x_4 x_1 + 2x_6 x_3 x_1 + x_4 x_3 x_1 - \\ &\quad x_3^2 x_1 - x_6 x_2 x_1 + x_5 x_2 x_1 - x_4 x_2 x_1 + x_3 x_2 x_1 - x_2^2 x_1 + x_5 x_1^2 - 3x_4 x_2^2 - x_3 x_1^2 - x_2 x_1^2 - x_3 x_2 - x_3 x_1 + x_6 \\ B(31)_{21} &= -x_6^2 x_4 + 2x_6 x_5 x_4 - x_5^2 x_4 - x_6 x_2^2 + x_5 x_2^2 - x_6^2 x_3 - x_5 x_3^2 - x_6 x_4 x_3 + x_5 x_4 x_3 + x_6 x_4 x_2 - \\ &\quad x_5 x_4 x_2 + x_6 x_3 x_2 - x_5 x_3 x_2 - x_4^2 - x_4 x_3 - x_6 x_2 + x_5 x_2 + x_4 x_2 - x_6 x_1 + x_5 x_1 + x_4 x_1 \\ B(31)_{22} &= x_6^2 x_4 - 2x_6 x_5 x_4 + x_5^2 x_4 + x_6 x_4^2 - x_5 x_4^2 + x_6^2 x_3 - 2x_6 x_5 x_3 + x_5^2 x_3 + x_6 x_4 x_3 - x_5 x_4 x_3 - x_6 x_4 x_2 + \\ &\quad x_5 x_4 x_2 - x_6 x_3 x_2 + x_5 x_3 x_2 - x_4 x_3 - x_3^2 + x_6 x_2 - x_5 x_2 + x_3 x_2 + x_6 x_1 - x_5 x_1 + x_3 x_1 \end{aligned}$$

$$\begin{aligned}
B(31)_{23} &= -x_6 x_4^2 x_3 + x_5 x_4^2 x_3 - x_4^3 x_3 - x_6 x_4 x_3^2 + x_5 x_4 x_3^2 - x_4^2 x_3^2 + x_6 x_4 x_3 x_2 - x_5 x_4 x_3 x_2 + 2 x_4^2 x_3 x_2 + \\
&x_6 x_3^2 x_2 - x_5 x_3^2 x_2 + 2 x_4 x_3^2 x_2 - x_4 x_3 x_2^2 - x_3^2 x_2^2 - x_6 x_4^2 x_1 + x_5 x_4^2 x_1 - x_4^3 x_1 - x_6 x_4 x_3 x_1 + x_5 x_4 x_3 x_1 - x_4^2 x_3 x_1 + \\
&x_6 x_4 x_2 x_1 - x_5 x_4 x_2 x_1 + 2 x_4^2 x_2 x_1 + x_6 x_3 x_2 x_1 - x_5 x_3 x_2 x_1 + 2 x_4 x_3 x_2 x_1 - x_4 x_2^2 x_1 - x_3 x_2^2 x_1 - x_6^3 + 2 x_6^2 x_5 - \\
&x_6 x_5^2 - x_6^2 x_4 + 2 x_6 x_5 x_4 + x_6^2 x_3 - x_6 x_3^2 + x_6^2 x_2 - 2 x_6 x_5 x_2 + x_6 x_4 x_2 + x_6 x_3 x_2 - x_4 x_3 x_2 - x_6 x_2^2 + x_3 x_2^2 - 2 x_6^2 x_1 + \\
&2 x_6 x_5 x_1 - x_5^2 x_1 - 3 x_6 x_4 x_1 + 2 x_5 x_4 x_1 + x_6 x_3 x_1 + 2 x_6 x_2 x_1 - 2 x_5 x_2 x_1 + x_3 x_2 x_1 - 2 x_4 x_1^2 + x_2 x_1^2 - x_6 x_3 + x_5 x_3 - \\
&x_4 x_3 + x_6 x_2 - x_5 x_2 + x_4 x_2 + x_3 x_2 - x_2^2 + x_6 + x_4 \\
B(31)_{24} &= -x_6 x_4^3 + x_5 x_4^3 - x_4^4 - x_6 x_4^2 x_3 + x_5 x_4^2 x_3 - x_4^3 x_3 + x_4^3 x_2 + x_4^2 x_3 x_2 + x_6 x_4 x_2^2 - x_5 x_4 x_2^2 + x_4^2 x_2^2 + \\
&x_6 x_3 x_2^2 - x_5 x_3 x_2^2 + x_4 x_3 x_2^2 - x_4 x_3^3 - x_3 x_3^3 + x_6^3 - 2 x_6^2 x_5 + x_6 x_5^2 + x_6^2 x_4 - x_6 x_4^2 - x_6^2 x_3 + 2 x_6 x_5 x_3 - x_6 x_4 x_3 + \\
&x_6 x_3^2 - x_6^2 x_2 - x_4^2 x_2 - x_6 x_3 x_2 + x_6 x_2^2 + x_3^2 + 2 x_6^2 x_1 - 4 x_6 x_5 x_1 + x_5^2 x_1 + 3 x_6 x_4 x_1 - x_4^2 x_1 - 2 x_6 x_3 x_1 + 2 x_5 x_3 x_1 + \\
&x_3^2 x_1 - x_6 x_2 x_1 + x_2^2 x_1 + x_6 x_1^2 - 2 x_5 x_1^2 + x_4 x_1^2 - x_3 x_1^2 - x_2 x_1^2 + x_6 x_3 - x_5 x_3 - x_3^2 - x_6 x_2 + x_5 x_2 + x_3 x_2 + x_3 x_1 - \\
&x_2 x_1 - x_6 - x_4 \\
B(31)_{25} &= -x_6^2 x_4^2 + x_5^2 x_4^2 - x_6 x_4^3 - x_5 x_4^3 - x_6^2 x_4 x_3 + x_5^2 x_4 x_3 - x_6 x_4 x_3^2 - x_5 x_4 x_3^2 + x_6^2 x_4 x_2 - x_5^2 x_4 x_2 + 2 x_6 x_4^2 x_2 + \\
&2 x_5 x_4^2 x_2 + x_6^2 x_3 x_2 - x_5^2 x_3 x_2 + 2 x_6 x_4 x_3 x_2 + 2 x_5 x_4 x_3 x_2 - x_6 x_4 x_2^2 - x_5 x_4 x_2^2 - x_6 x_3 x_2^2 - x_5 x_3 x_2^2 - 2 x_6 x_5 x_4 + \\
&x_6 x_4^2 - 2 x_6 x_5 x_3 + x_5 x_4 x_3 - x_4^2 x_3 - x_6 x_3^2 + x_5 x_3^2 - x_4 x_3^2 + 2 x_6 x_5 x_2 - 2 x_6 x_4 x_2 - x_5 x_4 x_2 + x_4 x_3 x_2 + x_3^2 x_2 + \\
&x_6 x_2^2 + x_5 x_2^2 + 2 x_6 x_5 x_1 + x_6 x_4 x_1 - 3 x_5 x_4 x_1 + 2 x_4^2 x_1 + 3 x_6 x_3 x_1 - 2 x_5 x_3 x_1 + 2 x_4 x_3 x_1 - x_6 x_2 x_1 + 2 x_5 x_2 x_1 - \\
&2 x_4 x_2 x_1 - x_3 x_2 x_1 - 2 x_6 x_2^2 + x_5 x_2^2 + x_3 x_2^2 - x_2 x_2^2 - x_1^3 + x_4 x_3 + x_3^2 - 2 x_3 x_2 - 2 x_3 x_1 + x_2 x_1 + x_1^2 \\
B(31)_{31} &= -x_6^2 x_4 + x_6 x_5 x_4 - x_6 x_4^2 + x_5 x_4^2 - x_6^2 x_3 + x_6 x_5 x_3 - x_6 x_4 x_3 + x_5 x_4 x_3 - x_6 x_3 - x_5 x_3 - x_4 x_3 - x_3^2 + \\
&x_4 x_2 - x_3 x_2 + x_4 x_1 - x_3 x_1 + x_1 \\
B(31)_{32} &= x_6^2 x_4 - x_6 x_5 x_4 + x_6 x_4^2 - x_5 x_4^2 + x_6^2 x_3 - x_6 x_5 x_3 + x_6 x_4 x_3 - x_5 x_4 x_3 - x_6 x_4 - x_5 x_4 - x_4^2 - x_4 x_3 - \\
&x_4 x_2 + x_3 x_2 - x_4 x_1 + x_3 x_1 + x_2 \\
B(31)_{33} &= -x_6 x_4^2 x_3 - x_4^3 x_3 - x_6 x_4 x_3^2 - x_4^2 x_3^2 + x_6 x_4 x_3 x_2 + x_4^2 x_3 x_2 + x_6 x_3^2 x_2 + x_4 x_3^2 x_2 - x_6 x_4^2 x_1 - x_4^3 x_1 - \\
&x_6 x_4 x_3 x_1 - x_4^2 x_3 x_1 + x_6 x_4 x_2 x_1 + x_4^2 x_2 x_1 + x_6 x_3 x_2 x_1 + x_4 x_3 x_2 x_1 + 2 x_6^2 x_5 + 2 x_6 x_5 x_4 + x_6^2 x_3 + x_6 x_4 x_3 + x_6^2 x_2 + \\
&x_6 x_4 x_2 + x_6 x_3 x_2 + 2 x_6 x_5 x_1 + 2 x_5 x_4 x_1 + x_6 x_3 x_1 - x_5 x_3 x_1 - x_3^2 x_1 + 2 x_6 x_2 x_1 + x_4 x_2 x_1 - x_3 x_2 x_1 - x_5 x_1^2 - x_4 x_1^2 - \\
&2 x_3 x_1^2 - x_2 x_1^2 - x_1^3 - x_6 x_3 - x_4 x_3 + x_3^2 + x_6 x_2 + x_4 x_2 + x_3 x_1 \\
B(31)_{34} &= -x_6 x_4^3 - x_4^4 - x_6 x_4^2 x_3 - x_4^3 x_3 + x_6 x_4 x_2^2 + x_4^2 x_2^2 + x_6 x_3 x_2^2 + x_4 x_3 x_2^2 + x_6^3 + x_6 x_5^2 + x_6^2 x_4 + 2 x_6 x_5 x_3 + \\
&x_6 x_3^2 + x_6 x_4 x_2 + x_6 x_2^2 + 2 x_6^2 x_1 + x_5^2 x_1 + 2 x_6 x_4 x_1 - x_5 x_4 x_1 - x_4^2 x_1 + 2 x_5 x_3 x_1 - x_4 x_3 x_1 + x_3^2 x_1 - x_5 x_2 x_1 - \\
&2 x_4 x_2 x_1 - x_3 x_2 x_1 - x_2^2 x_1 + x_6 x_2^2 - x_2 x_2^2 - x_5 x_3 + x_4 x_3 - x_3^2 + x_5 x_2 + 2 x_3 x_2 - x_6 - x_4 \\
B(31)_{35} &= -x_6^2 x_4^2 - x_6 x_5 x_4^2 - x_6 x_4^3 - x_5 x_4^3 - x_6^2 x_4 x_3 - x_5 x_4 x_3 - x_6 x_4 x_3^2 + x_6^2 x_4 x_2 + x_6 x_5 x_4 x_2 + \\
&x_6 x_4^2 x_2 + x_5 x_4^2 x_2 + x_6^2 x_3 x_2 + x_6 x_5 x_3 x_2 + x_6 x_4 x_3 x_2 + x_5 x_4 x_3 x_2 + x_6^2 x_4 + x_6 x_4^2 + x_6^2 x_3 - x_4^2 x_3 - x_6 x_3^2 + \\
&x_6 x_2 + 2 x_6 x_5 x_2 + x_6 x_3 x_2 - x_5^2 x_1 + x_6 x_4 x_1 - x_5 x_4 x_1 + 2 x_4^2 x_1 + 2 x_6 x_3 x_1 - x_5 x_3 x_1 + 2 x_4 x_3 x_1 - x_6 x_2 x_1 + x_3 x_2 x_1 - \\
&x_6 x_1^2 + x_3 x_1^2 + x_6 x_3 + x_5 x_3 - x_6 x_2 - x_5 x_2 - x_4 x_2 - 2 x_3 x_2 - x_2^2 - x_3 x_1 - x_2 x_1 + x_4 \\
B(31)_{41} &= x_6 \\
B(31)_{42} &= x_5 + 1 \\
B(31)_{43} &= -x_6 x_4 - x_4^2 + x_4 x_3 - x_6 x_2 - x_4 x_2 + x_4 x_1 \\
B(31)_{44} &= -x_5 x_4 + x_4^2 - x_4 x_3 - x_5 x_2 + x_4 x_2 - x_3 x_2 - x_3 - x_1 \\
B(31)_{45} &= x_6 x_4 + x_5 x_4 - x_4 x_2 - x_2^2 - x_4 x_1 - x_2 x_1 + x_3 \\
B(31)_{51} &= x_6^2 - x_5^2 + x_6 + 1 \\
B(31)_{52} &= -x_6^2 + x_5^2 + x_5 \\
B(31)_{53} &= x_6 x_4 x_3 + x_5 x_4 x_3 - x_6 x_3 x_2 - x_5 x_3 x_2 + x_6 x_4 x_1 + x_5 x_4 x_1 - x_6 x_2 x_1 - x_5 x_2 x_1 - x_5 x_4 + x_4 x_3 - \\
&x_5 x_2 - x_4 x_2 - x_2^2 + x_4 x_1 - x_3 - x_1 \\
B(31)_{54} &= x_6 x_4^2 + x_5 x_4^2 - x_6 x_2^2 - x_5 x_2^2 - x_6 x_4 + x_4^2 - x_6 x_2 + x_4 x_2 - x_4 x_1 - x_2 x_1 \\
B(31)_{55} &= x_6^2 x_4 + 2 x_6 x_5 x_4 + x_5^2 x_4 - x_6^2 x_2 - 2 x_6 x_5 x_2 - x_5^2 x_2 + x_6 x_4 + x_5 x_4 - x_4^2 + x_6 x_3 + x_5 x_3 - x_4 x_3 - \\
&x_4 x_2 - x_3 x_2 - x_6 x_1 - x_5 x_1 + x_3
\end{aligned}$$

Entries of $B(2^2)$.

$$\begin{aligned}
B(2^2)_{11} &= -x_6 x_4 - x_5 x_4 - x_6 x_3 - x_5 x_3 + x_4 x_2 + x_3 x_2 + x_4 x_1 + x_3 x_1 + x_2 + x_1 \\
B(2^2)_{12} &= x_5 x_4 - x_4^2 + x_6 x_3 + 2 x_4 x_3 - x_3^2 - x_4 x_2 - x_3 x_1 - x_2 \\
B(2^2)_{13} &= -x_6^3 - 2 x_6^2 x_5 - x_6 x_5^2 + x_6^2 x_4 + x_6 x_4^2 + x_6^2 x_3 - x_6 x_3^2 + x_6^2 x_2 + 2 x_6 x_5 x_2 - x_6 x_4 x_2 + x_6 x_3 x_2 - x_6 x_2^2 - \\
&x_5^2 x_1 - 2 x_6 x_4 x_1 + x_5 x_4 x_1 - x_4^2 x_1 + x_5 x_3 x_1 + x_2^2 x_1 + x_6 x_2 x_1 + x_5 x_2 x_1 - 2 x_3 x_2 x_1 + 2 x_6 x_1^2 + x_5 x_1^2 + x_4 x_1^2 - \\
&x_3 x_1^2 - x_2 x_1^2 - x_1^3 + x_6 x_3 + x_5 x_3 - x_4 x_3 - x_3^2 - x_6 x_2 - x_5 x_2 + x_2^2 + x_2 x_1 + x_6 + x_4
\end{aligned}$$

$$\begin{aligned}
B(2^2)_{14} &= x_6^3 + x_6x_5^2 - 2x_6x_5x_4 - x_6^2x_3 + 2x_6x_5x_3 - x_6x_4x_3 + x_6x_3^2 - x_6^2x_2 - 2x_6x_5x_2 + x_6x_4x_2 - x_6x_3x_2 + \\
&x_6x_2^2 + 2x_6x_5x_1 + x_6x_3x_1 - x_5x_3x_1 + x_4x_3x_1 - x_3^2x_1 - x_6x_2x_1 - x_5x_2x_1 + x_4x_2x_1 + x_2^2x_1 - x_6x_1^2 - x_6x_3 + x_4x_3 + \\
&x_6x_2 + x_4x_2 - x_3x_1 - x_2x_1 - x_6 \\
B(2^2)_{21} &= -x_4^2 - 2x_4x_3 - x_3^2 + x_4x_2 + x_3x_2 + x_4x_1 + x_3x_1 \\
B(2^2)_{22} &= -x_6x_4 + x_5x_4 + x_6x_3 - x_5x_3 + 2x_4x_3 - x_3x_2 - x_4x_1 - x_2 + x_1 \\
B(2^2)_{23} &= -2x_6x_5x_4 + x_6x_4^2 - 2x_6x_5x_3 + x_6x_4x_3 + 2x_6x_5x_2 - x_6x_4x_2 + 2x_6x_5x_1 - 2x_6x_4x_1 - x_6x_3x_1 + \\
&x_4x_3x_1 + x_3^2x_1 + x_6x_2x_1 + x_4x_2x_1 - x_2^2x_1 + x_6x_1^2 - x_3x_1^2 - x_2x_1^2 - x_4x_3 - x_3^2 - x_4x_2 + x_2^2 + x_3x_1 + x_2x_1 \\
B(2^2)_{24} &= x_6^3 - 2x_6^2x_5 + x_6x_5^2 + x_6^2x_4 - x_6x_4^2 - x_6^2x_3 + 2x_6x_5x_3 - x_6x_4x_3 + x_6x_3^2 - x_6^2x_2 - x_6x_3x_2 + x_6x_2^2 - \\
&x_5^2x_1 + 2x_6x_4x_1 + x_5x_4x_1 - x_4^2x_1 + x_6x_3x_1 - x_5x_3x_1 - x_5x_2x_1 - 2x_3x_2x_1 + x_2^2x_1 - 2x_6x_1^2 + x_5x_1^2 + x_4x_1^2 + \\
&x_3x_1^2 + x_2x_1^2 - x_1^3 - x_6x_3 + x_5x_3 + x_3^2 + x_6x_2 - x_5x_2 + x_4x_2 - x_2^2 - x_3x_1 - x_6 + x_4 \\
B(2^2)_{31} &= -x_6 - x_5 + 1 \\
B(2^2)_{32} &= x_6 \\
B(2^2)_{33} &= -x_6x_4 - x_5x_4 - x_6x_2 - x_5x_2 + 2x_4x_2 + x_3x_2 + x_4x_1 - x_3 - x_1 \\
B(2^2)_{34} &= x_5x_4 - x_4^2 + x_4x_3 + x_6x_2 - 2x_4x_2 - x_2^2 + x_2x_1 + x_3 \\
B(2^2)_{41} &= 0 \text{ (this is the only zero entry in all five matrices)} \\
B(2^2)_{42} &= x_6 - x_5 + 1 \\
B(2^2)_{43} &= -x_4^2 - x_4x_3 + 2x_4x_2 + x_3x_2 - x_2^2 + x_4x_1 - x_2x_1 \\
B(2^2)_{44} &= -x_6x_4 + x_5x_4 + x_4x_3 + x_6x_2 - x_5x_2 - x_3x_2 - x_4x_1 + x_2x_1 + x_3 - x_1
\end{aligned}$$

Entries of $B(21^2)$.

$$\begin{aligned}
B(21^2)_{11} &= x_5x_4^2 - x_5x_3^2 - x_5x_4x_2 + x_5x_3x_2 - x_5x_4x_1 + x_5x_3x_1 + x_6x_4 - x_5x_4 + x_4^2 - x_4x_3 - x_4x_2 + x_3x_2 + x_2 \\
B(21^2)_{12} &= x_6x_4^2 - x_6x_3^2 - x_6x_4x_2 + x_6x_3x_2 - x_6x_4x_1 + x_6x_3x_1 - x_4^2 + x_6x_3 - x_5x_3 + x_4x_3 + x_4x_2 - x_3x_2 + x_1 \\
B(21^2)_{13} &= -x_6x_4^3 + x_5x_4^3 + x_4^4 - x_4^3x_3 + x_6x_4x_3^2 - x_5x_4x_3^2 - x_4^2x_3^2 + x_4x_3^3 - 2x_4^3x_2 - x_6x_4x_3x_2 + x_5x_4x_3x_2 + \\
&2x_4^2x_3x_2 + x_6x_3^2x_2 - x_5x_3^2x_2 + x_6x_4x_2^2 - x_5x_4x_2^2 + x_4^2x_2^2 - x_6x_3x_2^2 + x_5x_3x_2^2 - x_4x_3x_2^2 + x_6x_4^2x_1 - x_5x_4^2x_1 - \\
&x_6x_4x_3x_1 + x_5x_4x_3x_1 + 2x_4^2x_3x_1 - 2x_4x_3^2x_1 + x_6x_4x_2x_1 - x_5x_4x_2x_1 - x_6x_3x_2x_1 + x_5x_3x_2x_1 - x_4^2x_1^2 + x_4x_3x_1^2 - \\
&x_6^3 + 2x_6^2x_5 - x_6x_5^2 + x_6x_4x_3 + x_4^2x_3 - x_6x_3^2 - x_3^3 + x_6x_4x_2 - x_4x_3x_2 + x_2^2x_2 - x_6x_2^2 - x_2^2x_1 + 2x_6x_5x_1 - x_5x_1 - \\
&x_6x_4x_1 - x_5x_4x_1 + x_4^2x_1 + x_5x_3x_1 - 2x_4x_3x_1 + x_2^2x_1 + x_5x_2x_1 - 2x_4x_2x_1 + x_3x_2x_1 + x_2^2x_1 + x_6x_1^2 - x_5x_1^2 + \\
&x_4x_1^2 - x_2x_1^2 - x_6x_3 + x_5x_3 + x_4x_3 - x_3^2 + x_6x_2 - x_5x_2 - x_3x_2 + x_3x_1 + x_6 + x_4 \\
B(21^2)_{14} &= x_6x_4^3 - x_5x_4^3 - x_4^4 + x_4^3x_3 - x_6x_4x_3^2 + x_5x_4x_3^2 + x_4^2x_3^2 - x_4x_3^3 - x_6x_4^2x_2 + x_5x_4^2x_2 + x_4^3x_2 + x_6x_4x_3x_2 - \\
&x_5x_4x_3x_2 - 2x_4^2x_3x_2 + x_4x_2^2x_2 + x_4^2x_2^2 - x_3x_2^2 - x_4x_3^2 + x_3x_2^3 - x_6x_4^2x_1 + x_5x_4^2x_1 + x_6x_4x_3x_1 - x_5x_4x_3x_1 - \\
&2x_4^2x_3x_1 + 2x_4x_3^2x_1 + x_4^2x_2x_1 - x_4x_3x_2x_1 - x_4x_2^2x_1 + x_3x_2^2x_1 + x_4^2x_1^2 - x_6x_4 - 2x_6x_5x_4 + x_6^2x_3 - \\
&x_6x_4x_3 - x_4^2x_3 + x_3^3 + x_6^2x_2 - 2x_6x_5x_2 + x_6x_3x_2 + x_4x_3x_2 - x_3^2x_2 - 2x_6x_5x_1 + x_5x_1 - x_6x_4x_1 - x_6x_3x_1 - x_5x_3x_1 + \\
&2x_4x_3x_1 - x_3^2x_1 + x_6x_2x_1 - x_3x_2x_1 + x_6x_1^2 + x_5x_1^2 + x_3x_2^2 - x_1^3 + x_6x_3 - x_4x_3 + x_3^2 - x_6x_2 + x_5x_2 - x_4x_2 + \\
&x_3x_2 + x_2^2 - x_3x_1 - x_4 \\
B(21^2)_{15} &= -x_6x_4^3 + x_5x_4^3 + x_4^4 - x_4^3x_3 + x_6x_4x_3^2 - x_5x_4x_3^2 - x_4^2x_3^2 + x_4x_3^3 + x_6x_4^2x_2 - x_5x_4^2x_2 - 2x_4^3x_2 - \\
&x_6x_4x_3x_2 + x_5x_4x_3x_2 + x_4^2x_3x_2 + x_3^2x_2 + x_4^2x_2^2 - x_3^2x_2^2 + x_6x_4^2x_1 - x_5x_4^2x_1 - x_6x_4x_3x_1 + x_5x_4x_3x_1 + 2x_4^2x_3x_1 - \\
&2x_4x_3^2x_1 + x_4^2x_2x_1 + x_4x_3x_2x_1 - 2x_4^2x_2x_1 - x_4x_2^2x_1 + x_3x_2^2x_1 - x_4^2x_1^2 + x_4x_3x_1^2 - x_4x_2x_1^2 + x_3x_2x_1^2 + x_6^2x_4 - \\
&x_6x_4^2 - x_6x_3^2 + 2x_6x_5x_3 + x_4^2x_3 - x_3^3 - x_6x_2 + x_6x_4x_2 - x_6x_3x_2 - x_4x_3x_2 + x_3^2x_2 - x_5x_1 - x_5x_4x_1 + x_4^2x_1 - \\
&x_6x_3x_1 - x_4x_3x_1 + x_3^2x_1 + x_6x_2x_1 + x_5x_2x_1 - x_4x_2x_1 + x_3x_2x_1 + x_6x_1^2 - x_2x_1^2 - x_6x_3 + x_5x_3 + x_4x_3 - x_3^2 + \\
&x_6x_2 - x_5x_2 - 2x_3x_2 + x_3x_1 + x_2x_1 + x_4 \\
B(21^2)_{21} &= x_6x_3 - x_5x_3 - x_4x_3 + x_3^2 + x_3x_2 - x_3x_1 + x_1 \\
B(21^2)_{22} &= x_6x_4 - x_5x_4 - x_4^2 + x_4x_3 + x_4x_2 - x_4x_1 + x_2 \\
B(21^2)_{23} &= -x_6^3 + 2x_6^2x_5 - x_6x_5^2 + 2x_6^2x_4 - 2x_6x_5x_4 - x_6x_4^2 - 2x_6^2x_3 + 2x_6x_5x_3 + 2x_6x_4x_3 - x_6x_3^2 - 2x_6^2x_2 + \\
&2x_6x_5x_2 + 2x_6x_4x_2 - 2x_6x_3x_2 - x_6x_2^2 + x_6^2x_1 - x_5^2x_1 - x_6x_4x_1 - x_5x_4x_1 + x_6x_3x_1 + x_5x_3x_1 + x_6x_2x_1 + x_5x_2x_1 - \\
&x_5x_1^2 - x_6x_3 + x_5x_3 + x_6x_2 - x_5x_2 - x_4x_2 + x_3x_2 + x_2^2 - x_2x_1 + x_6 + x_4 \\
B(21^2)_{24} &= x_6^3 - 2x_6^2x_5 + x_6x_5^2 - 2x_6^2x_4 + 2x_6x_5x_4 + x_6x_4^2 + 2x_6^2x_3 - 2x_6x_5x_3 - 2x_6x_4x_3 + x_6x_3^2 + 2x_6^2x_2 - \\
&2x_6x_5x_2 - 2x_6x_4x_2 + 2x_6x_3x_2 + x_6x_2^2 - 2x_6^2x_1 + 2x_6x_5x_1 + x_6x_4x_1 + x_5x_4x_1 + x_4^2x_1 - 2x_6x_3x_1 - x_4x_3x_1 - \\
&x_6x_2x_1 - x_5x_2x_1 - 2x_4x_2x_1 + x_3x_2x_1 + x_2^2x_1 + x_6x_1^2 + x_4x_2^2 - x_2x_1^2 - x_4x_3 + x_3x_2 - x_6 \\
B(21^2)_{25} &= -x_6^3 + 2x_6^2x_5 - x_6x_5^2 + 2x_6^2x_4 - 2x_6x_5x_4 - x_6x_4^2 - 2x_6^2x_3 + 2x_6x_5x_3 + 2x_6x_4x_3 - x_6x_3^2 - 2x_6^2x_2 + \\
&2x_6x_5x_2 + 2x_6x_4x_2 - 2x_6x_3x_2 - x_6x_2^2 + 2x_6^2x_1 - 2x_6x_5x_1 - 2x_6x_4x_1 + x_6x_3x_1 + x_5x_3x_1 + x_4x_3x_1 - x_3^2x_1 + \\
&2x_6x_2x_1 - x_3x_2x_1 - x_5x_1^2 - x_4x_2^2 + 2x_3x_1^2 + x_2x_1^2 - x_1^3 - x_3x_1 + x_6
\end{aligned}$$

$$\begin{aligned}
B(21^2)_{31} &= x_6 x_5 - x_5^2 + 1 \\
B(21^2)_{32} &= x_6^2 - x_6 x_5 \\
B(21^2)_{33} &= -x_6^2 x_4 + 2 x_6 x_5 x_4 - x_5^2 x_4 + x_6 x_4^2 - x_5 x_4^2 - x_6 x_4 x_3 + x_5 x_4 x_3 - x_6^2 x_2 + 2 x_6 x_5 x_2 - x_5^2 x_2 - x_6 x_4 x_2 + \\
&x_5 x_4 x_2 + x_6 x_4 x_1 - x_5 x_4 x_1 + x_4^2 + x_6 x_3 - x_5 x_3 - x_4 x_3 - x_4 x_2 + x_3 x_2 + x_6 x_1 - x_5 x_1 \\
B(21^2)_{34} &= x_6^2 x_4 - 2 x_6 x_5 x_4 + x_5^2 x_4 - x_6 x_4^2 + x_5 x_4^2 + x_6 x_4 x_3 - x_5 x_4 x_3 + x_6 x_2^2 - x_5 x_2^2 - x_6 x_4 x_1 + x_5 x_4 x_1 + \\
&x_6 x_4 - x_6 x_3 + x_5 x_3 - x_6 x_2 - x_4 x_1 + x_2 x_1 \\
B(21^2)_{35} &= -x_6^2 x_4 + 2 x_6 x_5 x_4 - x_5^2 x_4 + x_6 x_4^2 - x_5 x_4^2 - x_6 x_4 x_3 + x_5 x_4 x_3 - x_6 x_4 x_2 + x_5 x_4 x_2 - x_6 x_3 x_2 + \\
&x_5 x_3 x_2 + x_6 x_4 x_1 - x_5 x_4 x_1 + x_6 x_2 x_1 - x_5 x_2 x_1 + x_5 x_4 + x_6 x_3 - x_5 x_3 - x_5 x_2 - x_4 x_2 + x_2^2 + x_3 - x_1 \\
B(21^2)_{41} &= x_6 x_5 x_4 + x_5^2 x_4 - x_6 x_5 x_3 - x_5^2 x_3 - x_5 x_4 x_2 + x_5 x_3 x_2 - x_5 x_4 x_1 + x_5 x_3 x_1 + x_6 x_4 + x_4^2 - x_6 x_3 - \\
&x_4 x_3 - x_5 x_2 + x_5 x_1 - x_4 x_1 + x_3 x_1 \\
B(21^2)_{42} &= x_6^2 x_4 + x_6 x_5 x_4 - x_6^2 x_3 - x_6 x_5 x_3 - x_6 x_4 x_2 + x_6 x_3 x_2 - x_6 x_4 x_1 + x_6 x_3 x_1 - x_6 x_4 + x_6 x_3 + x_4 x_3 - \\
&x_3^2 - x_6 x_2 + x_6 x_1 + x_4 x_1 - x_3 x_1 \\
B(21^2)_{43} &= -x_6^2 x_4^2 + x_5^2 x_4^2 + x_6 x_4^3 + x_5 x_4^3 + x_6^2 x_4 x_3 - x_5^2 x_4 x_3 - 2 x_6 x_4^2 x_3 - 2 x_5 x_4^2 x_3 + x_6 x_4 x_3^2 + x_5 x_4 x_3^2 - \\
&x_6^2 x_4 x_2 + x_5^2 x_4 x_2 - 2 x_5 x_4^2 x_2 - x_4^3 x_2 + x_6^2 x_3 x_2 - x_5^2 x_3 x_2 + 2 x_5 x_4 x_3 x_2 + 2 x_4^2 x_3 x_2 - x_4 x_3^2 x_2 + x_6 x_4 x_2^2 - x_5 x_4 x_2^2 + \\
&x_4^2 x_2^2 - x_6 x_3 x_2^2 + x_5 x_3 x_2^2 - x_4 x_3 x_2^2 + 2 x_6 x_4^2 x_1 - x_4^3 x_1 - 2 x_6 x_4 x_3 x_1 + 2 x_4^2 x_3 x_1 - x_4 x_3^2 x_1 + x_6 x_4 x_2 x_1 - x_5 x_4 x_2 x_1 - \\
&x_6 x_3 x_2 x_1 + x_5 x_3 x_2 x_1 - x_4^2 x_1 + x_4 x_3 x_1^2 - x_6^2 x_4 + 2 x_6 x_5 x_4 + x_6^2 x_3 - 2 x_6 x_5 x_3 + x_6 x_4 x_3 + x_5 x_4 x_3 - x_6 x_3^2 - x_5 x_3^2 + \\
&x_6^2 x_2 - x_5 x_4 x_2 - x_4^2 x_2 + x_6 x_3 x_2 + x_3^2 x_2 + x_6 x_2^2 - x_5 x_2^2 + x_4 x_2^2 - x_6^2 x_1 - x_6 x_4 x_1 + 2 x_5 x_4 x_1 + 2 x_4^2 x_1 - x_5 x_3 x_1 - \\
&4 x_4 x_3 x_1 + 2 x_3^2 x_1 - 3 x_6 x_2 x_1 + x_5 x_2 x_1 - 3 x_4 x_2 x_1 + x_3 x_2 x_1 + 2 x_6 x_1^2 + x_4 x_1^2 + x_2 x_1^2 - x_1^3 + x_4 x_3 - x_3^2 + x_4 x_2 - \\
&2 x_3 x_2 + x_3 x_1 - x_2 x_1 + x_1^2 \\
B(21^2)_{44} &= x_6^2 x_4^2 - x_5^2 x_4^2 - x_6 x_4^3 - x_5 x_4^3 - x_6^2 x_4 x_3 + x_5^2 x_4 x_3 + 2 x_6 x_4^2 x_3 + 2 x_5 x_4^2 x_3 - x_6 x_4 x_3^2 - x_5 x_4 x_3^2 - \\
&x_6 x_4^2 x_2 + x_5 x_4^2 x_2 + x_3^2 x_2 + x_6 x_4 x_3 x_2 - x_5 x_4 x_3 x_2 - 2 x_4^2 x_3 x_2 + x_4 x_3^2 x_2 + x_6 x_4 x_2^2 + x_5 x_4 x_2^2 - x_6 x_3 x_2^2 - x_5 x_3 x_2^2 - \\
&x_4 x_3^3 + x_3 x_3^2 - 2 x_6 x_4^2 x_1 + x_4^3 x_1 + 2 x_6 x_4 x_3 x_1 - 2 x_4^2 x_3 x_1 + x_4 x_3^2 x_1 + x_4^2 x_2 x_1 - x_4 x_3 x_2 x_1 - x_4 x_2^2 x_1 + x_3 x_2^2 x_1 + \\
&x_4^2 x_1^2 - x_4 x_3 x_1^2 + 2 x_6^2 x_5 - x_6^2 x_4 - x_6 x_3 - x_5 x_4 x_3 + x_6 x_3^2 + x_5 x_3^2 - x_6^2 x_2 + x_5 x_4 x_2 + x_4^2 x_2 - x_6 x_3 x_2 - x_3^2 x_2 - \\
&x_2^3 - 4 x_6 x_5 x_1 + 2 x_6 x_4 x_1 - 2 x_5 x_4 x_1 - 2 x_4^2 x_1 + x_6 x_3 x_1 + x_5 x_3 x_1 + 4 x_4 x_3 x_1 - 2 x_3^2 x_1 + 2 x_6 x_2 x_1 + 2 x_4 x_2 x_1 - \\
&x_3 x_2 x_1 + x_2^2 x_1 + 2 x_5 x_2^2 - 2 x_4 x_1^2 - x_2 x_1^2 + x_6 x_3 - x_6 x_2 - x_4 x_2 + 2 x_3 x_2 - 2 x_3 x_1 + x_2 x_1 \\
B(21^2)_{45} &= -x_6^2 x_4^2 + x_5^2 x_4^2 + x_6 x_4^3 + x_5 x_4^3 + x_6^2 x_4 x_3 - x_5^2 x_4 x_3 - 2 x_6 x_4^2 x_3 - 2 x_5 x_4^2 x_3 + x_6 x_4 x_3^2 + x_5 x_4 x_3^2 - \\
&2 x_5 x_4^2 x_2 - x_4^3 x_2 - x_6 x_4 x_3 x_2 + x_5 x_4 x_3 x_2 + 2 x_4^2 x_3 x_2 + x_6 x_3^2 x_2 + x_5 x_3^2 x_2 - x_4 x_3^2 x_2 + x_4^2 x_2^2 - x_3^2 x_2^2 + 2 x_6 x_4 x_2 x_1 - \\
&x_4^3 x_1 - 2 x_6 x_4 x_3 x_1 + 2 x_4^2 x_3 x_1 - x_4 x_3^2 x_1 + x_6 x_4 x_2 x_1 + x_5 x_4 x_2 x_1 - x_6 x_3 x_2 x_1 - x_5 x_3 x_2 x_1 + x_4 x_3 x_2 x_1 - x_3^2 x_2 x_1 - \\
&x_4 x_2^2 x_1 + x_3 x_2^2 x_1 - x_4^2 x_1^2 + x_4 x_3 x_1^2 - x_4 x_2 x_1^2 + x_3 x_2 x_1^2 + x_6^3 + x_6 x_5^2 - x_6^2 x_4 + x_5 x_4 x_3 - x_5 x_3^2 - 2 x_6 x_5 x_2 + x_6 x_4 x_2 - \\
&x_5 x_4 x_2 - x_4^2 x_2 + x_3^2 x_2 + x_6 x_2^2 + x_4 x_2^2 + x_3 x_2^2 - 2 x_6^2 x_1 - x_5^2 x_1 + x_6 x_4 x_1 + 2 x_5 x_4 x_1 + 2 x_4^2 x_1 - x_5 x_3 x_1 - 4 x_4 x_3 x_1 + \\
&2 x_3^2 x_1 + 2 x_5 x_2 x_1 - 3 x_4 x_2 x_1 - 2 x_2^2 x_1 + x_6 x_1^2 + x_4 x_1^2 - x_3 x_1^2 + x_2 x_1^2 + x_5 x_3 - x_5 x_2 + x_4 x_2 - 3 x_3 x_2 + x_2^2 + x_3 x_1 - \\
&x_6 + x_4 \\
B(21^2)_{51} &= x_6 x_5 - x_5^2 + x_6 - x_5 \\
B(21^2)_{52} &= x_6^2 - x_6 x_5 - x_6 + x_5 + 1 \\
B(21^2)_{53} &= -x_6^2 x_4 + 2 x_6 x_5 x_4 - x_5^2 x_4 + x_6 x_4^2 - x_5 x_4^2 - x_6 x_4 x_3 + x_5 x_4 x_3 - x_6^2 x_2 + 2 x_6 x_5 x_2 - x_5^2 x_2 - x_6 x_4 x_2 + \\
&x_5 x_4 x_2 + x_6 x_4 x_1 - x_5 x_4 x_1 + x_6 x_3 - x_5 x_3 + x_4 x_2 - x_2^2 + x_6 x_1 - x_5 x_1 - x_4 x_1 + x_2 x_1 \\
B(21^2)_{54} &= x_6^2 x_4 - 2 x_6 x_5 x_4 + x_5^2 x_4 - x_6 x_4^2 + x_5 x_4^2 + x_6 x_4 x_3 - x_5 x_4 x_3 + x_6 x_2^2 - x_5 x_2^2 - x_6 x_4 x_1 + x_5 x_4 x_1 + \\
&x_5 x_4 - x_6 x_3 + x_5 x_3 - x_4 x_3 - x_5 x_2 + x_3 x_2 + x_3 - x_1 \\
B(21^2)_{55} &= -x_6^2 x_4 + 2 x_6 x_5 x_4 - x_5^2 x_4 + x_6 x_4^2 - x_5 x_4^2 - x_6 x_4 x_3 + x_5 x_4 x_3 - x_6 x_4 x_2 + x_5 x_4 x_2 - x_6 x_3 x_2 + \\
&x_5 x_3 x_2 + x_6 x_4 x_1 - x_5 x_4 x_1 + x_6 x_2 x_1 - x_5 x_2 x_1 + x_6 x_4 - x_4^2 + x_6 x_3 - x_5 x_3 - x_6 x_2 + x_4 x_2
\end{aligned}$$

Entries of $B(1^4)$.

$$\begin{aligned}
B(1^4)_{11} &= -x_6 x_4 + x_5 x_4 + x_4^2 + x_6 x_3 - x_5 x_3 - 2 x_4 x_3 + x_3^2 - x_4 x_2 + x_3 x_2 + x_4 x_1 - x_3 x_1 - x_2 + x_1 \\
B(1^4)_{12} &= x_6^3 - 2 x_6^2 x_5 + x_6 x_5^2 - 2 x_6^2 x_4 + 2 x_6 x_5 x_4 + x_6 x_4^2 + 2 x_6^2 x_3 - 2 x_6 x_5 x_3 - 2 x_6 x_4 x_3 + x_6 x_3^2 + 2 x_6^2 x_2 - \\
&2 x_6 x_5 x_2 - 2 x_6 x_4 x_2 + 2 x_6 x_3 x_2 + x_6 x_2^2 - 3 x_6^2 x_1 + 4 x_6 x_5 x_1 - x_5^2 x_1 + 4 x_6 x_4 x_1 - 2 x_5 x_4 x_1 - x_4^2 x_1 - 4 x_6 x_3 x_1 + \\
&2 x_5 x_3 x_1 + 2 x_4 x_3 x_1 - x_3^2 x_1 - 4 x_6 x_2 x_1 + 2 x_5 x_2 x_1 + 2 x_4 x_2 x_1 - 2 x_3 x_2 x_1 - x_2^2 x_1 + 3 x_6 x_1^2 - 2 x_5 x_1^2 - 2 x_4 x_1^2 + \\
&2 x_3 x_1^2 + 2 x_2 x_1^2 - x_1^3 - x_6 x_3 + x_5 x_3 + x_4 x_3 - x_3^2 + x_6 x_2 - x_5 x_2 - x_4 x_2 + x_2^2 + x_3 x_1 - x_2 x_1 - x_6 + x_4 \\
B(1^4)_{21} &= x_6 - x_5 + 1 \\
B(1^4)_{22} &= -x_6 x_4 + x_5 x_4 + x_4^2 - x_4 x_3 + x_6 x_2 - x_5 x_2 - 2 x_4 x_2 + x_3 x_2 + x_2^2 + x_4 x_1 - x_2 x_1 + x_3 - x_1
\end{aligned}$$

```

Ground<[x]> := PolynomialRing(RationalField(), 6, "grevlex");

mat4 := Matrix(Ground, [
[
- x[6]*x[4] - x[5]*x[4] - x[4]^2 - x[6]*x[3] - x[5]*x[3] - 2*x
[4]*x[3] - x[3]^2 - x[4]*x[2] - x[3]*x[2] - x[4]*x[1] - x[3]*x
[1] + x[2] + x[1],
- x[6]^3 - 2*x[6]^2*x[5] - x[6]*x[5]^2 - 2*x[6]^2*x[4] - 2*x[6]*x
[5]*x[4] - x[6]*x[4]^2 - 2*x[6]^2*x[3] - 2*x[6]*x[5]*x[3] - 2*
x[6]*x[4]*x[3] - x[6]*x[3]^2 - 2*x[6]^2*x[2] - 2*x[6]*x[5]*x
[2] - 2*x[6]*x[4]*x[2] - 2*x[6]*x[3]*x[2] - x[6]*x[2]^2 - 3*x
[6]^2*x[1] - 4*x[6]*x[5]*x[1] - x[5]^2*x[1] - 4*x[6]*x[4]*x
[1] - 2*x[5]*x[4]*x[1] - x[4]^2*x[1] - 4*x[6]*x[3]*x[1] - 2*x
[5]*x[3]*x[1] - 2*x[4]*x[3]*x[1] - x[3]^2*x[1] - 4*x[6]*x[2]*x
[1] - 2*x[5]*x[2]*x[1] - 2*x[4]*x[2]*x[1] - 2*x[3]*x[2]*x[1]
- x[2]^2*x[1] - 3*x[6]*x[1]^2 - 2*x[5]*x[1]^2 - 2*x[4]*x[1]^2
- 2*x[3]*x[1]^2 - 2*x[2]*x[1]^2 - x[1]^3 + x[6]*x[3] + x[5]*x
[3] + x[4]*x[3] + x[3]^2 - x[6]*x[2] - x[5]*x[2] - x[4]*x[2] -
x[2]^2 + x[3]*x[1] - x[2]*x[1] + x[6] + x[4]
],
[
- x[6] - x[5] + 1,
- x[6]*x[4] - x[5]*x[4] - x[4]^2 - x[4]*x[3] - x[6]*x[2] - x[5]*x
[2] - 2*x[4]*x[2] - x[3]*x[2] - x[2]^2 - x[4]*x[1] - x[2]*x[1]
- x[3] - x[1]
]
]);
mat31 := Matrix(Ground, [
[
- x[6]^2*x[4] + x[6]*x[5]*x[4] - x[6]*x[4]^2 + x[5]*x[4]^2 - x
[6]^2*x[3] + x[6]*x[5]*x[3] + x[6]*x[3]^2 - x[5]*x[3]^2 + x
[6]*x[4]*x[1] - x[5]*x[4]*x[1] + x[6]*x[3]*x[1] - x[5]*x[3]*x
[1] - x[6]*x[4] - x[5]*x[4] + x[4]*x[2] + x[4]*x[1] + x[2],
x[6]^2*x[4] - x[6]*x[5]*x[4] + x[6]*x[4]^2 - x[5]*x[4]^2 + x[6]^2*
x[3] - x[6]*x[5]*x[3] - x[6]*x[3]^2 + x[5]*x[3]^2 - x[6]*x[4]*x
[1] + x[5]*x[4]*x[1] - x[6]*x[3]*x[1] + x[5]*x[3]*x[1] - x[6]*x
[3] - x[5]*x[3] + x[3]*x[2] + x[3]*x[1] + x[1],

```

```

- x[6]*x[4]^2*x[3] - x[4]^3*x[3] - x[6]*x[4]*x[3]^2 + x[4]*x[3]^3
  + x[6]*x[4]*x[3]*x[2] + x[4]^2*x[3]*x[2] + x[6]*x[3]^2*x[2]
- x[3]^3*x[2] - x[6]*x[4]^2*x[1] - x[4]^3*x[1] - x[6]*x[4]*x
  [3]*x[1] + x[4]^2*x[3]*x[1] + 2*x[4]*x[3]^2*x[1] + x[6]*x[4]*x
  [2]*x[1] + x[4]^2*x[2]*x[1] + x[6]*x[3]*x[2]*x[1] - x[4]*x[3]*
  x[2]*x[1] - 2*x[3]^2*x[2]*x[1] + x[4]^2*x[1]^2 + x[4]*x[3]*x
  [1]^2 - x[4]*x[2]*x[1]^2 - x[3]*x[2]*x[1]^2 + 2*x[6]^2*x[5] - 
  x[6]^2*x[4] + 2*x[6]*x[5]*x[4] - x[6]*x[4]^2 - 2*x[6]*x[5]*x
  [3] + x[6]*x[4]*x[3] - x[6]^2*x[1] - x[6]*x[4]*x[1] + 2*x[5]*x
  [4]*x[1] - x[4]^2*x[1] + 2*x[6]*x[3]*x[1] - x[5]*x[3]*x[1] + x
  [4]*x[3]*x[1] - x[3]*x[2]*x[1] + x[6]*x[1]^2 - x[5]*x[1]^2 - x
  [2]*x[1]^2 - x[6]*x[3] - x[4]*x[3] + x[6]*x[2] + x[4]*x[2] - x
  [3]*x[2] - x[2]*x[1] ,
- x[6]*x[4]^3 - x[4]^4 - x[6]*x[4]^2*x[3] + x[4]^2*x[3]^2 + x[6]*
  x[4]*x[2]^2 + x[4]^2*x[2]^2 + x[6]*x[3]*x[2]^2 - x[3]^2*x[2]^2
  + x[4]^3*x[1] + x[4]^2*x[3]*x[1] - x[4]*x[2]^2*x[1] - x[3]*x
  [2]^2*x[1] + x[6]^3 + x[6]*x[5]^2 - 2*x[6]*x[5]*x[4] - x
  [6]^2*x[3] + 2*x[6]*x[5]*x[3] - x[6]*x[4]*x[3] + x[6]*x[3]^2 -
  x[6]^2*x[2] - 2*x[6]*x[5]*x[2] + x[6]*x[4]*x[2] - x[6]*x[3]*x
  [2] + x[6]*x[2]^2 + x[6]^2*x[1] + x[5]^2*x[1] + 3*x[6]*x[4]*x
  [1] - x[5]*x[4]*x[1] - 2*x[6]*x[3]*x[1] + 2*x[5]*x[3]*x[1] - x
  [4]*x[3]*x[1] + x[3]^2*x[1] + x[6]*x[2]*x[1] - x[5]*x[2]*x[1]
  - x[3]*x[2]*x[1] - x[6]*x[1]^2 + x[4]*x[1]^2 - x[3]*x[1]^2 - x
  [1]^3 - x[5]*x[3] - x[3]^2 + x[5]*x[2] - x[4]*x[2] + x[3]*x[2]
  - x[2]^2 - x[6] - x[4] ,
- x[6]^2*x[4]^2 - x[6]*x[5]*x[4]^2 - x[6]*x[4]^3 - x[5]*x[4]^3 -
  x[6]^2*x[4]*x[3] - x[6]*x[5]*x[4]*x[3] + x[6]*x[4]*x[3]^2 + x
  [5]*x[4]*x[3]^2 + x[6]^2*x[4]*x[2] + x[6]*x[5]*x[4]*x[2] + x
  [6]*x[4]^2*x[2] + x[5]*x[4]^2*x[2] + x[6]^2*x[3]*x[2] + x[6]*x
  [5]*x[3]*x[2] - x[6]*x[3]^2*x[2] - x[5]*x[3]^2*x[2] + x[6]*x
  [4]^2*x[1] + x[5]*x[4]^2*x[1] + x[6]*x[4]*x[3]*x[1] + x[5]*x
  [4]*x[3]*x[1] - x[6]*x[4]*x[2]*x[1] - x[5]*x[4]*x[2]*x[1] - x
  [6]*x[3]*x[2]*x[1] - x[5]*x[3]*x[2]*x[1] - x[6]^3 - 2*x[6]^2*x
  [5] - x[6]*x[5]^2 + x[6]^2*x[4] + x[6]*x[4]^2 + x[6]^2*x[3] - 
  x[6]*x[4]*x[3] - x[4]^2*x[3] - 2*x[6]*x[3]^2 + x[3]^3 + x
  [6]^2*x[2] + 2*x[6]*x[5]*x
x[2] - x[6]*x[4]*x[2] + x[6]*x[3]*x[2] - x[6]*x[2]^2 + x[6]^2*x[1]
  + 2*x[6]*x[5]*x[1] + 2*x[4]^2*x[1] + 2*x[6]*x[3]*x[1] + x[4]*x
  [3]*x[1] - x[3]^2*x[1] - x[6]*x[2]*x[1] + x[5]*x[2]*x[1] - x
  [4]*x[2]*x[1] + x[3]*x[2]*x[1] - x[2]^2*x[1] + x[5]*x[1]^2 - 3*
  x[4]*x[1]^2 - x[3]*x[1]^2 - x[2]*x[1]^2 - x[3]*x[2] - x[3]*x[1]
  + x[6]
],
[

```

$$\begin{aligned}
& -x[6]^2*x[4] + 2*x[6]*x[5]*x[4] - x[5]^2*x[4] - x[6]*x[4]^2 + x \\
& [5]*x[4]^2 - x[6]^2*x[3] + 2*x[6]*x[5]*x[3] - x[5]^2*x[3] - x \\
& [6]*x[4]*x[3] + x[5]*x[4]*x[3] + x[6]*x[4]*x[2] - x[5]*x[4]*x \\
& [2] + x[6]*x[3]*x[2] - x[5]*x[3]*x[2] - x[4]^2 - x[4]*x[3] - x \\
& [6]*x[2] + x[5]*x[2] + x[4]*x[2] - x[6]*x[1] + x[5]*x[1] + x \\
& [4]*x[1], \\
& x[6]^2*x[4] - 2*x[6]*x[5]*x[4] + x[5]^2*x[4] + x[6]*x[4]^2 - x[5]* \\
& x[4]^2 + x[6]^2*x[3] - 2*x[6]*x[5]*x[3] + x[5]^2*x[3] + x[6]*x \\
& [4]*x[3] - x[5]*x[4]*x[3] - x[6]*x[4]*x[2] + x[5]*x[4]*x[2] - x \\
& [6]*x[3]*x[2] + x[5]*x[3]*x[2] - x[4]*x[3] - x[3]^2 + x[6]*x[2] \\
& - x[5]*x[2] + x[3]*x[2] + x[6]*x[1] - x[5]*x[1] + x[3]*x[1], \\
& -x[6]*x[4]^2*x[3] + x[5]*x[4]^2*x[3] - x[4]^3*x[3] - x[6]*x[4]*x \\
& [3]^2 + x[5]*x[4]*x[3]^2 - x[4]^2*x[3]^2 + x[6]*x[4]*x[3]*x[2] \\
& - x[5]*x[4]*x[3]*x[2] + 2*x[4]^2*x[3]*x[2] + x[6]*x[3]^2*x[2] \\
& - x[5]*x[3]^2*x[2] + 2*x[4]*x[3]^2*x[2] - x[4]*x[3]*x[2]^2 - \\
& x[3]^2*x[2]^2 - x[6]*x[4]^2*x[1] + x[5]*x[4]^2*x[1] - x[4]^3*x \\
& [1] - x[6]*x[4]*x[3]*x[1] + x[5]*x[4]*x[3]*x[1] - x[4]^2*x[3]* \\
& x[1] + x[6]*x[4]*x[2]*x[1] - x[5]*x[4]*x[2]*x[1] + 2*x[4]^2*x \\
& [2]*x[1] + x[6]*x[3]*x[2]*x[1] - x[5]*x[3]*x[2]*x[1] + 2*x[4]* \\
& x[3]*x[2]*x[1] - x[4]*x[2]^2*x[1] - x[3]*x[2]^2*x[1] - x[6]^3 \\
& + 2*x[6]^2*x[5] - x[6]*x[5]^2 - x[6]^2*x[4] + 2*x[6]*x[5]*x[4] \\
& + x[6]^2*x[3] - x[6]*x[3]^2 + x[6]^2*x[2] - 2*x[6]*x[5]*x[2] \\
& + x[6]*x[4]*x[2] + x[6]*x[3]*x[2] - x[4]*x[3]*x[2] - x[6]*x \\
& [2]^2 + x[3]*x[2]^2 - 2*x[6]^2*x[1] + 2*x[6]*x[5]*x[1] - x \\
& [5]^2*x[1] - 3*x[6]*x[4]*x[1] + 2*x[5]*x[4]*x[1] + x[6]*x[3]*x \\
& [1] + 2*x[6]*x[2]*x[1] - 2*x[5]*x[2]*x[1] + x[3]*x[2]*x[1] - \\
& 2*x[4]*x[1]^2 + x[2]*x[1]^2 - x[6]*x[3] + x[5]*x[3] - x[4]*x \\
& [3] + x[6]*x[2] - x[5]*x[2] + x[4]*x[2] + x[3]*x[2] - x[2]^2 + \\
& x[6] + x[4], \\
& -x[6]*x[4]^3 + x[5]*x[4]^3 - x[4]^4 - x[6]*x[4]^2*x[3] + x[5]*x \\
& [4]^2*x[3] - x[4]^3*x[3] + x[4]^3*x[2] + x[4]^2*x[3]*x[2] + x \\
& [6]*x[4]*x[2]^2 - x[5]*x[4]*x[2]^2 + x[4]^2*x[2]^2 + x[6]*x \\
& [3]*x[2]^2 - x[5]*x[3]*x[2]^2 + x[4]*x[3]*x[2]^2 - x[4]*x[2]^3 \\
& - x[3]*x[2]^3 + x[6]^3 - 2*x[6]^2*x[5] + x[6]*x[5]^2 + x \\
& [6]^2*x[4] - x[6]*x[4]^2 - x[6]^2*x[3] + 2*x[6]*x[5]*x[3] - x \\
& [6]*x[4]*x[3] + x[6]*x[3]^2 - x[6]^2*x[2] - x[4]^2*x[2] - x \\
& [6]*x[3]*x[2] + x[6]*x[2]^2 + x[2]^3 + 2*x[6]^2*x[1] - 4*x[6]* \\
& x[5]*x[1] + x[5]^2*x[1] + 3*x[6]*x[4]*x[1] - x[4]^2*x[1] - 2*x \\
& [6]*x[3]*x[1] + 2*x[5]*x[3]*x[1] + x[3]^2*x[1] - x[6]*x[2]*x \\
& [1] + x[2]^2*x[1] + x[6]*x[1]^2 - 2*x[5]*x[1]^2 + x[4]*x[1]^2 \\
& - x[3]*x[1]^2 - x[2]*x[1]^2 + x[6]*x[3] - x[5]*x[3] - x[3]^2 - \\
& x[6]*x[2] + x[5]*x[2] + x[3]*x[2] + x[3]*x[1] - x[2]*x[1] - \\
& x[6] - x[4],
\end{aligned}$$

```

- x[6]^2*x[4]^2 + x[5]^2*x[4]^2 - x[6]*x[4]^3 - x[5]*x[4]^3 - x
[6]^2*x[4]*x[3] + x[5]^2*x[4]*x[3] - x[6]*x[4]^2*x[3] - x[5]*x
[4]^2*x[3] + x[6]^2*x[4]*x[2] - x[5]^2*x[4]*x[2] + 2*x[6]*x
[4]^2*x[2] + 2*x[5]*x[4]^2*x[2] + x[6]^2*x[3]*x[2] - x[5]^2*x
[3]*x[2] + 2*x[6]*x[4]*x[3]*x[2] + 2*x[5]*x[4]*x[3]*x[2] - x
[6]*x[4]*x[2]^2 - x[5]*x[4]*x[2]^2 - x[6]*x[3]*x[2]^2 - x[5]*x
[3]*x[2]^2 - 2*x[6]*x[5]*x[4] + x[6]*x[4]^2 - 2*x[6]*x[5]*x[3]
+ x[5]*x[4]*x[3] - x[4]^2*x[3] - x[6]*x[3]^2 + x[5]*x[3]^2 -
x[4]*x[3]^2 + 2*x[6]*x[5]*x[2] - 2*x[6]*x[4]*x[2] - x[5]*x[4]*
x[2] + x[4]*x[3]*x[2] + x[3]^2*x[2] + x[6]*x[2]^2 + x[5]*x
[2]^2 + 2*x[6]*x[5]*x[1] + x[6]*x[4]*x[1] - 3*x[5]*x[4]*x[1] +
2*x[4]^2*x[1] + 3*x[6]*x[3]*x[1] - 2*x[5]*x[3]*x[1] + 2*x[4]*
x[3]*x[1] - x[6]*x[2]*x[1] + 2*x[5]*x[2]*x[1] - 2*x[4]*x[2]*x
[1] - x[3]*x[2]*x[1] - 2*x[6]*x[1]^2 + x[5]*x[1]^2 + x[3]*x
[1]^2 - x[2]*x[1]^2 - x[1]^3 + x[4]*x[3] + x[3]^2 - 2*x[3]*x
[2] - 2*x[3]*x[1] + x[2]*x[1] + x[1]^2
],
[
- x[6]^2*x[4] + x[6]*x[5]*x[4] - x[6]*x[4]^2 + x[5]*x[4]^2 - x
[6]^2*x[3] + x[6]*x[5]*x[3] - x[6]*x[4]*x[3] + x[5]*x[4]*x[3]
- x[6]*x[3] - x[5]*x[3] - x[4]*x[3] - x[3]^2 + x[4]*x[2] - x
[3]*x[2] + x[4]*x[1] - x[3]*x[1] + x[1],
x[6]^2*x[4] - x[6]*x[5]*x[4] + x[6]*x[4]^2 - x[5]*x[4]^2 + x[6]^2*
x[3] - x[6]*x[5]*x[3] + x[6]*x[4]*x[3] - x[5]*x[4]*x[3] - x[6]*
x[4] - x[5]*x[4] - x[4]^2 - x[4]*x[3] - x[4]*x[2] + x[3]*x[2] -
x[4]*x[1] + x[3]*x[1] + x[2],
- x[6]*x[4]^2*x[3] - x[4]^3*x[3] - x[6]*x[4]*x[3]^2 - x[4]^2*x
[3]^2 + x[6]*x[4]*x[3]*x[2] + x[4]^2*x[3]*x[2] + x[6]*x[3]^2*x
[2] + x[4]*x[3]^2*x[2] - x[6]*x[4]^2*x[1] - x[4]^3*x[1] - x
[6]*x[4]*x[3]*x[1] - x[4]^2*x[3]*x[1] + x[6]*x[4]*x[2]*x[1] +
x[4]^2*x[2]*x[1] + x[6]*x[3]*x[2]*x[1] + x[4]*x[3]*x[2]*x[1] +
2*x[6]^2*x[5] + 2*x[6]*x[5]*x[4] + x[6]^2*x[3] + x[6]*x[4]*x
[3] + x[6]^2*x[2] + x[6]*x[4]*x[2] + x[6]*x[3]*x[2] + 2*x[6]*
x[5]*x[1] + 2*x[5]*x[4]*x[1] + x[6]*x[3]*x[1] - x[5]*x[3]*x[1]
- x[3]^2*x[1] + 2*x[6]*x[2]*x[1] + x[4]*x[2]*x[1] - x[3]*x
[2]*x[1] - x[5]*x[1]^2 - x[4]*x[1]^2 - 2*x[3]*x[1]^2 - x[2]*x
[1]^2 - x[1]^3 - x[6]*x[3] - x[4]*x[3] + x[3]^2 + x[6]*x[2] +
x[4]*x[2] + x[3]*x[1],
- x[6]*x[4]^3 - x[4]^4 - x[6]*x[4]^2*x[3] - x[4]^3*x[3] + x[6]*x
[4]*x[2]^2 + x[4]^2*x[2]^2 + x[6]*x[3]*x[2]^2 + x[4]*x[3]*x
[2]^2 + x[6]^3 + x[6]*x[5]^2 + x[6]^2*x[4] + 2*x[6]*x[5]*x[3]
+ x[6]*x[3]^2 + x[6]*x[4]*x[2] + x[6]*x[2]^2 + 2*x[6]^2*x[1] +
x[5]^2*x[1] + 2*x[6]*x[4]*x[1] - x[5]*x[4]*x[1] - x[4]^2*x[1]
+ 2*x[5]*x[3]*x[1] - x[4]*x[3]*x[1] + x[3]^2*x[1] - x[5]*x
[2]*x[1] - 2*x[4]*x[2]*x[1] - x[3]*x[2]*x[1] - x[2]^2*x[1] + x
[6]*x[1]^2 - x[2]*x[1]^2 - x[5]*x[3] + x[4]*x[3] - x[3]^2 + x
[5]*x[2] + 2*x[3]*x[2] - x[6] - x[4],
]

```

```

- x[6]^2*x[4]^2 - x[6]*x[5]*x[4]^2 - x[6]*x[4]^3 - x[5]*x[4]^3 -
x[6]^2*x[4]*x[3] - x[6]*x[5]*x[4]*x[3] - x[6]*x[4]^2*x[3] - x
[5]*x[4]^2*x[3] + x[6]^2*x[4]*x[2] + x[6]*x[5]*x[4]*x[2] + x
[6]*x[4]^2*x[2] + x[5]*x[4]^2*x[2] + x[6]^2*x[3]*x[2] + x[6]*x
[5]*x[3]*x[2] + x[6]*x[4]*x[3]*x[2] + x[5]*x[4]*x[3]*x[2] + x
[6]^2*x[4] + x[6]*x[4]^2 + x[6]^2*x[3] - x[4]^2*x[3] - x[6]*x
[3]^2 - x[4]*x[3]^2 + x[6]^2*x[2] + 2*x[6]*x[5]*x[2] + x[6]*x
[3]*x[2] - x[5]^2*x[1] + x[6]*x[4]*x[1] - x[5]*x[4]*x[1] + 2*x
[4]^2*x[1] + 2*x[6]*x[3]*x[1] - x[5]*x[3]*x[1] + 2*x[4]*x[3]*x
[1] - x[6]*x[2]*x[1] + x[3]*x[2]*x[1] - x[6]*x[1]^2 + x[3]*x
[1]^2 + x[6]*x[3] + x[5]*x[3] - x[6]*x[2] - x[5]*x[2] - x[4]*x
[2] - 2*x[3]*x[2] - x[2]^2 - x[3]*x[1] - x[2]*x[1] + x[4]
],
[
x[6],
x[5] + 1,
- x[6]*x[4] - x[4]^2 + x[4]*x[3] - x[6]*x[2] - x[4]*x[2] + x[4]*x
[1],
- x[5]*x[4] + x[4]^2 - x[4]*x[3] - x[5]*x[2] + x[4]*x[2] - x[3]*x
[2] - x[3] - x[1],
x[6]*x[4] + x[5]*x[4] - x[4]*x[2] - x[2]^2 - x[4]*x[1] - x[2]*x[1]
+ x[3]
],
[
x[6]^2 - x[5]^2 + x[6] + 1,
- x[6]^2 + x[5]^2 + x[5],
x[6]*x[4]*x[3] + x[5]*x[4]*x[3] - x[6]*x[3]*x[2] - x[5]*x[3]*x[2]
+ x[6]*x[4]*x[1] + x[5]*x[4]*x[1] - x[6]*x[2]*x[1] - x[5]*x[2]*
x[1] - x[5]*x[4] + x[4]*x[3] - x[5]*x[2] - x[4]*x[2] - x[2]^2 +
x[4]*x[1] - x[3] - x[1],
x[6]*x[4]^2 + x[5]*x[4]^2 - x[6]*x[2]^2 - x[5]*x[2]^2 - x[6]*x[4]
+ x[4]^2 - x[6]*x[2] + x[4]*x[2] - x[4]*x[1] - x[2]*x[1],
x[6]^2*x[4] + 2*x[6]*x[5]*x[4] + x[5]^2*x[4] - x[6]^2*x[2] - 2*x
[6]*x[5]*x[2] - x[5]^2*x[2] + x[6]*x[4] + x[5]*x[4] - x[4]^2 +
x[6]*x[3] + x[5]*x[3] - x[4]*x[3] - x[4]*x[2] - x[3]*x[2] - x
[6]*x[1] - x[5]*x[1] + x[3]
]
]);
mat22 := Matrix(Ground,[
[
- x[6]*x[4] - x[5]*x[4] - x[6]*x[3] - x[5]*x[3] + x[4]*x[2] + x
[3]*x[2] + x[4]*x[1] + x[3]*x[1] + x[2] + x[1],
x[5]*x[4] - x[4]^2 + x[6]*x[3] + 2*x[4]*x[3] - x[3]^2 - x[4]*x[2]
- x[3]*x[1] - x[2],

```

```

- x[6]^3 - 2*x[6]^2*x[5] - x[6]*x[5]^2 + x[6]^2*x[4] + x[6]*x
[4]^2 + x[6]^2*x[3] - x[6]*x[3]^2 + x[6]^2*x[2] + 2*x[6]*x[5]*
x[2] - x[6]*x[4]*x[2] + x[6]*x[3]*x[2] - x[6]*x[2]^2 - x[5]^2*
x[1] - 2*x[6]*x[4]*x[1] + x[5]*x[4]*x[1] - x[4]^2*x[1] + x[5]*
x[3]*x[1] + x[3]^2*x[1] + x[6]*x[2]*x[1] + x[5]*x[2]*x[1] - 2*
x[3]*x[2]*x[1] + 2*x[6]*x[1]^2 + x[5]*x[1]^2 + x[4]*x[1]^2 - x
[3]*x[1]^2 - x[2]*x[1]^2 - x[1]^3 + x[6]*x[3] + x[5]*x[3] - x
[4]*x[3] - x[3]^2 - x[6]*x[2] - x[5]*x[2] + x[2]^2 + x[2]*x[1]
+ x[6] + x[4] ,
x[6]^3 + x[6]*x[5]^2 - 2*x[6]*x[5]*x[4] - x[6]^2*x[3] + 2*x[6]*x
[5]*x[3] - x[6]*x[4]*x[3] + x[6]*x[3]^2 - x[6]^2*x[2] - 2*x[6]*x
[5]*x[2] + x[6]*x[4]*x[2] - x[6]*x[3]*x[2] + x[6]*x[2]^2 + 2*x
[6]*x[5]*x[1] + x[6]*x[3]*x[1] - x[5]*x[3]*x[1] + x[4]*x[3]*x
[1] - x[3]^2*x[1] - x[6]*x[2]*x[1] - x[5]*x[2]*x[1] + x[4]*x
[2]*x[1] + x[2]^2*x[1] - x[6]*x[1]^2 - x[6]*x[3] + x[4]*x[3] +
x[6]*x[2] + x[4]*x[2] - x[3]*x[1] - x[2]*x[1] - x[6]
],
[
- x[4]^2 - 2*x[4]*x[3] - x[3]^2 + x[4]*x[2] + x[3]*x[2] + x[4]*x
[1] + x[3]*x[1] ,
- x[6]*x[4] + x[5]*x[4] + x[6]*x[3] - x[5]*x[3] + 2*x[4]*x[3] - x
[3]*x[2] - x[4]*x[1] - x[2] + x[1] ,
- 2*x[6]*x[5]*x[4] + x[6]*x[4]^2 - 2*x[6]*x[5]*x[3] + x[6]*x[4]*x
[3] + 2*x[6]*x[5]*x[2] - x[6]*x[4]*x[2] + 2*x[6]*x[5]*x[1] -
2*x[6]*x[4]*x[1] - x[6]*x[3]*x[1] + x[4]*x[3]*x[1] + x[3]^2*x
[1] + x[6]*x[2]*x[1] + x[4]*x[2]*x[1] - x[2]^2*x[1] + x[6]*x
[1]^2 - x[3]*x[1]^2 - x[2]*x[1]^2 - x[4]*x[3] - x[3]^2 - x[4]*
x[2] + x[2]^2 + x[3]*x[1] + x[2]*x[1] ,
x[6]^3 - 2*x[6]^2*x[5] + x[6]*x[5]^2 + x[6]^2*x[4] - x[6]*x[4]^2 -
x[6]^2*x[3] + 2*x[6]*x[5]*x[3] - x[6]*x[4]*x[3] + x[6]*x[3]^2 -
x[6]^2*x[2] - x[6]*x[3]*x[2] + x[6]*x[2]^2 - x[5]^2*x[1] + 2*
x[6]*x[4]*x[1] + x[5]*x[4]*x[1] - x[4]^2*x[1] + x[6]*x[3]*x[1] -
x[5]*x[3]*x[1] - x[5]*x[2]*x[1] - 2*x[3]*x[2]*x[1] + x[2]^2*x
[1] - 2*x[6]*x[1]^2 + x[5]*x[1]^2 + x[4]*x[1]^2 + x[3]*x[1]^2 +
x[2]*x[1]^2 - x[1]^3 - x[6]*x[3] + x[5]*x[3] + x[3]^2 + x[6]*
x[2] - x[5]*x[2] + x[4]*x[2] - x[2]^2 - x[3]*x[1] - x[6] + x[4]
],
[
- x[6] - x[5] + 1,
x[6] ,
- x[6]*x[4] - x[5]*x[4] - x[6]*x[2] - x[5]*x[2] + 2*x[4]*x[2] + x
[3]*x[2] + x[4]*x[1] - x[3] - x[1] ,
x[5]*x[4] - x[4]^2 + x[4]*x[3] + x[6]*x[2] - 2*x[4]*x[2] - x[2]^2
+ x[2]*x[1] + x[3]
],
[
0,
x[6] - x[5] + 1,
- x[4]^2 - x[4]*x[3] + 2*x[4]*x[2] + x[3]*x[2] - x[2]^2 + x[4]*x
[1] - x[2]*x[1] ,

```

```

- x[6]*x[4] + x[5]*x[4] + x[4]*x[3] + x[6]*x[2] - x[5]*x[2] - x
[3]*x[2] - x[4]*x[1] + x[2]*x[1] + x[3] - x[1]
]

]) ;

mat211 := Matrix(Ground,
[x[5]*x[4]^2 - x[5]*x[3]^2 - x[5]*x[4]*x[2] + x[5]*x[3]*x[2] - x
[5]*x[4]*x[1] + x[5]*x[3]*x[1] + x[6]*x[4] - x[5]*x[4] + x[4]^2
- x[4]*x[3] - x[4]*x[2] + x[3]*x[2] + x[2],
x[6]*x[4]^2 - x[6]*x[3]^2 - x[6]*x[4]*x[2] + x[6]*x[3]*x[2] - x
[6]*x[4]*x[1] + x[6]*x[3]*x[1] - x[4]^2 + x[6]*x[3] - x[5]*x[3]
+ x[4]*x[3] + x[4]*x[2] - x[3]*x[2] + x[1],
- x[6]*x[4]^3 + x[5]*x[4]^3 + x[4]^4 - x[4]^3*x[3] + x[6]*x[4]*x
[3]^2 - x[5]*x[4]*x[3]^2 - x[4]^2*x[3]^2 + x[4]*x[3]^3 - 2*x
[4]^3*x[2] - x[6]*x[4]*x[3]*x[2] + x[5]*x[4]*x[3]*x[2] + 2*x
[4]^2*x[3]*x[2] + x[6]*x[3]^2*x[2] - x[5]*x[3]^2*x[2] + x[6]*x
[4]*x[2]^2 - x[5]*x[4]*x[2]^2 + x[4]^2*x[2]^2 - x[6]*x[3]*x
[2]^2 + x[5]*x[3]*x[2]^2 - x[4]*x[3]*x[2]^2 + x[6]*x[4]^2*x[1]
- x[5]*x[4]^2*x[1] - x[6]*x[4]*x[3]*x[1] + x[5]*x[4]*x[3]*x
[1] + 2*x[4]^2*x[3]*x[1] - 2*x[4]*x[3]^2*x[1] + x[6]*x[4]*x
[2]*x[1] - x[5]*x[4]*x[2]*x[1] - x[6]*x[3]*x[2]*x[1] + x[5]*x
[3]*x[2]*x[1] - x[4]^2*x[1]^2 + x[4]*x[3]*x[1]^2 - x[6]^3 + 2*
x[6]^2*x[5] - x[6]*x[5]^2 + x[6]*x[4]*x[3] + x[4]^2*x[3] - x
[6]*x[3]^2 - x[3]^3 + x[6]*x[4]*x[2] - x[4]*x[3]*x[2] + x
[3]^2*x[2] - x[6]^2*x[1] + 2*x[6]*x[5]*x[1] - x
[5]^2*x[1] - x[6]*x[4]*x[1] - x[5]*x[4]*x[1] + x[4]^2*x[1] + x
[5]*x[3]*x[1] - 2*x[4]*x[3]*x[1] + x[3]^2*x[1] + x[5]*x[2]*x
[1] - 2*x[4]*x[2]*x[1] + x[3]*x[2]*x[1] + x[2]^2*x[1] + x[6]*x
[1]^2 - x[5]*x[1]^2 + x[4]*x[1]^2 - x[2]*x[1]^2 - x[6]*x[3] +
x[5]*x[3] + x[4]*x[3] - x[3]^2 + x[6]*x[2] - x[5]*x[2] - x[3]*
x[2] + x[3]*x[1] + x[6] + x[4],
x[6]*x[4]^3 - x[5]*x[4]^3 - x[4]^4 + x[4]^3*x[3] - x[6]*x[4]*x
[3]^2 + x[5]*x[4]*x[3]^2 + x[4]^2*x[3]^2 - x[4]*x[3]^3 - x[6]*x
[4]^2*x[2] + x[5]*x[4]^2*x[2] + x[4]^3*x[2] + x[6]*x[4]*x[3]*x
[2] - x[5]*x[4]*x[3]*x[2] - 2*x[4]^2*x[3]*x[2] + x[4]*x[3]^2*x
[2] + x[4]^2*x[2]^2 - x[3]^2*x[2]^2 - x[4]*x[2]^3 + x[3]*x[2]^3
- x[6]*x[4]^2*x[1] + x[5]*x[4]^2*x[1] + x[6]*x[4]*x[3]*x[1] -
x[5]*x[4]*x[3]*x[1] - 2*x[4]^2*x[3]*x[1] + 2*x[4]*x[3]^2*x[1]
+ x[4]^2*x[2]*x[1] - x[4]*x[3]*x[2]*x[1] - x[4]*x[2]^2*x[1] + x
[3]*x[2]^2*x[1] + x[4]^2*x[1]^2 - x[4]*x[3]*x[1]^2 - x[6]^2*x
[4] + 2*x[6]*x[5]*x[4] + x[6]^2*x[3] - x[6]*x[4]*x[3] - x[4]^2*
x[3] + x[3]^3 + x[6]^2*x[2] - 2*x[6]*x[5]*x[2] + x[6]*x[3]*x[2]
+ x[4]*x[3]*x[2] - x[3]^2*x[2] - 2*x[6]*x[5]*x[1] + x[5]^2*x
[1] - x[6]*x[4]*x[1] - x[6]*x[3]*x[1] - x[5]*x[3]*x[1] + 2*x
[4]*x[3]*x[1] - x[3]^2*x[1] + x[6]*x[2]*x[1] - x[3]*x[2]*x[1] +
x[6]*x[1]^2 + x[5]*x[1]^2 + x[3]*x[1]^2 - x[1]^3 + x[6]*x[3] -
x[5]*x[3] - x[4]*x[3] + x[3]^2 - x[6]*x[2] + x[5]*x[2] - x[4]*
x[2] + x[3]*x[2] + x[2]^2 - x[3]*x[1] - x[4],

```

```

- x[6]*x[4]^3 + x[5]*x[4]^3 + x[4]^4 - x[4]^3*x[3] + x[6]*x[4]*x
[3]^2 - x[5]*x[4]*x[3]^2 - x[4]^2*x[3]^2 + x[4]*x[3]^3 + x[6]*x
[4]^2*x[2] - x[5]*x[4]^2*x[2] - 2*x[4]^3*x[2] - x[6]*x[4]*x
[3]*x[2] + x[5]*x[4]*x[3]*x[2] + x[4]^2*x[3]*x[2] + x[3]^3*x
[2] + x[4]^2*x[2]^2 - x[3]^2*x[2]^2 + x[6]*x[4]^2*x[1] - x[5]*x
[4]^2*x[1] - x[6]*x[4]*x[3]*x[1] + x[5]*x[4]*x[3]*x[1] + 2*x
[4]^2*x[3]*x[1] - 2*x[4]*x[3]^2*x[1] + x[4]^2*x[2]*x[1] + x
[4]*x[3]*x[2]*x[1] - 2*x[3]^2*x[2]*x[1] - x[4]*x[2]^2*x[1] + x
[3]*x[2]^2*x[1] - x[4]^2*x[1]^2 + x[4]*x[3]*x[1]^2 - x[4]*x
[2]*x[1]^2 + x[3]*x[2]*x[1]^2 + x[6]^2*x[4] - x[6]*x[4]^2 - x
[6]^2*x[3] + 2*x[6]*x[5]*x[3] + x[4]^2*x[3] - x[3]^3 - x[6]^2*
x[2] + x[6]*x[4]*x[2] - x[6]*x[3]*x[2] - x[4]*x[3]*x[2] + x
[3]^2*x[2] - x[5]^2*x[1] - x[5]*x[4]*x[1] + x[4]^2*x[1] - x
[6]*x[3]*x[1] - x[4]*x[3]*x[1] + x[3]^2*x[1] + x[6]*x[2]*x[1]
+ x[5]*x[2]*x[1] - x[4]*x[2]*x[1] + x[3]*x[2]*x[1] + x[6]*x
[1]^2 - x[2]*x[1]^2 - x[6]*x[3] + x[5]*x[3] + x[4]*x[3] - x
[3]^2 + x[6]*x[2] - x[5]*x[2] - 2*x[3]*x[2] + x[3]*x[1] + x
[2]*x[1] + x[4]
],
[
x[6]*x[3] - x[5]*x[3] - x[4]*x[3] + x[3]^2 + x[3]*x[2] - x[3]*x[1]
+ x[1],
x[6]*x[4] - x[5]*x[4] - x[4]^2 + x[4]*x[3] + x[4]*x[2] - x[4]*x[1]
+ x[2],
- x[6]^3 + 2*x[6]^2*x[5] - x[6]*x[5]^2 + 2*x[6]^2*x[4] - 2*x[6]*x
[5]*x[4] - x[6]*x[4]^2 - 2*x[6]^2*x[3] + 2*x[6]*x[5]*x[3] + 2*
x[6]*x[4]*x[3] - x[6]*x[3]^2 - 2*x[6]^2*x[2] + 2*x[6]*x[5]*x
[2] + 2*x[6]*x[4]*x[2] - 2*x[6]*x[3]*x[2] - x[6]*x[2]^2 + x
[6]^2*x[1] - x[5]^2*x[1] - x[6]*x[4]*x[1] - x[5]*x[4]*x[1] + x
[6]*x[3]*x[1] + x[5]*x[3]*x[1] + x[6]*x[2]*x[1] + x[5]*x[2]*x
[1] - x[5]*x[1]^2 - x[6]*x[3] + x[5]*x[3] + x[6]*x[2] - x[5]*x
[2] - x[4]*x[2] + x[3]*x[2] + x[2]^2 - x[2]*x[1] + x[6] + x
[4], x[6]^3 - 2*x[6]^2*x[5] + x[6]*x[5]^2 - 2*x[6]^2*x[4] + 2*
x[6]*x[5]*x[4] + x[6]*x[4]^2 + 2*x[6]^2*x[3] - 2*x[6]*x[5]*x
[3] - 2*x[6]*x[4]*x[3] + x[6]*x[3]^2 + 2*x[6]^2*x[2] - 2*x[6]*x
[5]*x[2] - 2*x[6]*x[4]*x[2] + 2*x[6]*x[3]*x[2] + x[6]*x[2]^2
- 2*x[6]^2*x[1] + 2*x[6]*x[5]*x[1] + x[6]*x[4]*x[1] + x[5]*x
[4]*x[1] + x[4]^2*x[1] - 2*x[6]*x[3]*x[1] - x[4]*x[3]*x[1] - x
[6]*x[2]*x[1] - x[5]*x[2]*x[1] - 2*x[4]*x[2]*x[1] + x[3]*x[2]*x
[1] + x[2]^2*x[1] + x[6]*x[1]^2 + x[4]*x[1]^2 - x[2]*x[1]^2 - x
[4]*x[3] + x[3]*x[2] - x[6],
- x[6]^3 + 2*x[6]^2*x[5] - x[6]*x[5]^2 + 2*x[6]^2*x[4] - 2*x[6]*x
[5]*x[4] - x[6]*x[4]^2 - 2*x[6]^2*x[3] + 2*x[6]*x[5]*x[3] + 2*
x[6]*x[4]*x[3] - x[6]*x[3]^2 - 2*x[6]^2*x[2] + 2*x[6]*x[5]*x
[2] + 2*x[6]*x[4]*x[2] - 2*x[6]*x[3]*x[2] - x[6]*x[2]^2 + 2*x
[6]^2*x[1] - 2*x[6]*x[5]*x[1] - 2*x[6]*x[4]*x[1] + x[6]*x[3]*x
[1] + x[5]*x[3]*x[1] + x[4]*x[3]*x[1] - x[3]^2*x[1] + 2*x[6]*x
[2]*x[1] - x[3]*x[2]*x[1] - x[5]*x[1]^2 - x[4]*x[1]^2 + 2*x
[3]*x[1]^2 + x[2]*x[1]^2 - x[1]^3 - x[3]^2 + x[3]*x[1] + x[6]

```

```

] ,
[
x[6]*x[5] - x[5]^2 + 1,
x[6]^2 - x[6]*x[5],
- x[6]^2*x[4] + 2*x[6]*x[5]*x[4] - x[5]^2*x[4] + x[6]*x[4]^2 - x
[5]*x[4]^2 - x[6]*x[4]*x[3] + x[5]*x[4]*x[3] - x[6]^2*x[2] +
2*x[6]*x[5]*x[2] - x[5]^2*x[2] - x[6]*x[4]*x[2] + x[5]*x[4]*x
[2] + x[6]*x[4]*x[1] - x[5]*x[4]*x[1] + x[4]^2 + x[6]*x[3] - x
[5]*x[3] - x[4]*x[3] - x[4]*x[2] + x[3]*x[2] + x[6]*x[1] - x
[5]*x[1],
x[6]^2*x[4] - 2*x[6]*x[5]*x[4] + x[5]^2*x[4] - x[6]*x[4]^2 + x[5]*
x[4]^2 + x[6]*x[4]*x[3] - x[5]*x[4]*x[3] + x[6]*x[2]^2 - x[5]*x
[2]^2 - x[6]*x[4]*x[1] + x[5]*x[4]*x[1] + x[6]*x[4] - x[6]*x[3]
+ x[5]*x[3] - x[6]*x[2] - x[4]*x[1] + x[2]*x[1], -
x[6]^2*x[4] + 2*x[6]*x[5]*x[4] - x[5]^2*x[4] + x[6]*x[4]^2 - x[5]*
x[4]^2 - x[6]*x[4]*x[3] + x[5]*x[4]*x[3] - x[6]*x[4]*x[2] + x
[5]*x[4]*x[2] - x[6]*x[3]*x[2] + x[5]*x[3]*x[2] + x[6]*x[4]*x
[1] - x[5]*x[4]*x[1] + x[6]*x[2]*x[1] - x[5]*x[2]*x[1] + x[5]*x
[4] + x[6]*x[3] - x[5]*x[3] - x[5]*x[2] - x[4]*x[2] + x[2]^2 +
x[3] - x[1]
],
[
x[6]*x[5]*x[4] + x[5]^2*x[4] - x[6]*x[5]*x[3] - x[5]^2*x[3] - x
[5]*x[4]*x[2] + x[5]*x[3]*x[2] - x[5]*x[4]*x[1] + x[5]*x[3]*x
[1] + x[6]*x[4] + x[4]^2 - x[6]*x[3] - x[4]*x[3] - x[5]*x[2] +
x[5]*x[1] - x[4]*x[1] + x[3]*x[1],
x[6]^2*x[4] + x[6]*x[5]*x[4] - x[6]^2*x[3] - x[6]*x[5]*x[3] - x
[6]*x[4]*x[2] + x[6]*x[3]*x[2] - x[6]*x[4]*x[1] + x[6]*x[3]*x
[1] - x[6]*x[4] + x[6]*x[3] + x[4]*x[3] - x[3]^2 - x[6]*x[2] +
x[6]*x[1] + x[4]*x[1] - x[3]*x[1],

```

$$\begin{aligned}
& -x[6]^2*x[4]^2 + x[5]^2*x[4]^2 + x[6]*x[4]^3 + x[5]*x[4]^3 + x \\
& [6]^2*x[4]*x[3] - x[5]^2*x[4]*x[3] - 2*x[6]*x[4]^2*x[3] - 2*x \\
& [5]*x[4]^2*x[3] + x[6]*x[4]*x[3]^2 + x[5]*x[4]*x[3]^2 - x \\
& [6]^2*x[4]*x[2] + x[5]^2*x[4]*x[2] - 2*x[5]*x[4]^2*x[2] - x \\
& [4]^3*x[2] + x[6]^2*x[3]*x[2] - x[5]^2*x[3]*x[2] + 2*x[5]*x \\
& [4]*x[3]*x[2] + 2*x[4]^2*x[3]*x[2] - x[4]*x[3]^2*x[2] + x[6]* \\
& x[4]*x[2]^2 - x[5]*x[4]*x[2]^2 + x[4]^2*x[2]^2 - x[6]*x[3]*x \\
& [2]^2 + x[5]*x[3]*x[2]^2 - x[4]*x[3]*x[2]^2 + 2*x[6]*x[4]^2*x \\
& [1] - x[4]^3*x[1] - 2*x[6]*x[4]*x[3]*x[1] + 2*x[4]^2*x[3]*x[1] \\
& - x[4]*x[3]^2*x[1] + x[6]*x[4]*x[2]*x[1] - x[5]*x[4]*x[2]*x \\
& [1] - x[6]*x[3]*x[2]*x[1] + x[5]*x[3]*x[2]*x[1] - x[4]^2*x \\
& [1]^2 + x[4]*x[3]*x[1]^2 - x[6]^2*x[4] + 2*x[6]*x[5]*x[4] + x \\
& [6]^2*x[3] - 2*x[6]*x[5]*x[3] + x[6]*x[4]*x[3] + x[5]*x[4]*x \\
& [3] - x[6]*x[3]^2 - x[5]*x[3]^2 + x[6]^2*x[2] - x[5]*x[4]*x[2] \\
& - x[4]^2*x[2] + x[6]*x[3]*x[2] + x[3]^2*x[2] + x[6]*x[2]^2 - \\
& x[5]*x[2]^2 + x[4]*x[2]^2 - x[6]^2*x[1] - x[6]*x[4]*x[1] + 2*x \\
& [5]*x[4]*x[1] + 2*x[4]^2*x[1] - x[5]*x[3]*x[1] - 4*x[4]*x[3]*x \\
& [1] + 2*x[3]^2*x[1] - 3*x[6]*x[2]*x[1] + x[5]*x[2]*x[1] - 3*x \\
& [4]*x[2]*x[1] + x[3]*x[2]*x[1] + 2*x[6]*x[1]^2 + x[4]*x[1]^2 \\
& + x[2]*x[1]^2 - x[1]^3 + x[4]*x[3] - x[3]^2 + x[4]*x[2] - 2*x \\
& [3]*x[2] + x[3]*x[1] - x[2]*x[1] + x[1]^2, \\
& x[6]^2*x[4]^2 - x[5]^2*x[4]^2 - x[6]*x[4]^3 - x[5]*x[4]^3 - x \\
& [6]^2*x[4]*x[3] + x[5]^2*x[4]*x[3] + 2*x[6]*x[4]^2*x[3] + 2*x \\
& [5]*x[4]^2*x[3] - x[6]*x[4]*x[3]^2 - x[5]*x[4]*x[3]^2 - x[6]*x \\
& [4]^2*x[2] + x[5]*x[4]^2*x[2] + x[4]^3*x[2] + x[6]*x[4]*x[3]*x \\
& [2] - x[5]*x[4]*x[3]*x[2] - 2*x[4]^2*x[3]*x[2] + x[4]*x[3]^2*x \\
& [2] + x[6]*x[4]*x[2]^2 + x[5]*x[4]*x[2]^2 - x[6]*x[3]*x[2]^2 - \\
& x[5]*x[3]*x[2]^2 - x[4]*x[2]^3 + x[3]*x[2]^3 - 2*x[6]*x[4]^2*x \\
& [1] + x[4]^3*x[1] + 2*x[6]*x[4]*x[3]*x[1] - 2*x[4]^2*x[3]*x[1] \\
& + x[4]*x[3]^2*x[1] + x[4]^2*x[2]*x[1] - x[4]*x[3]*x[2]*x[1] - x \\
& [4]*x[2]^2*x[1] + x[3]*x[2]^2*x[1] + x[4]^2*x[1]^2 - x[4]*x[3]* \\
& x[1]^2 + 2*x[6]^2*x[5] - x[6]*x[4]^2 - x[6]^2*x[3] - x[5]*x[4]* \\
& x[3] + x[6]*x[3]^2 + x[5]*x[3]^2 - x[6]^2*x[2] + x[5]*x[4]*x \\
& [2] + x[4]^2*x[2] - x[6]*x[3]*x[2] - x[3]^2*x[2] - x[2]^3 - 4*x \\
& [6]*x[5]*x[1] + 2*x[6]*x[4]*x[1] - 2*x[5]*x[4]*x[1] - 2*x[4]^2*x \\
& [1] + x[6]*x[3]*x[1] + x[5]*x[3]*x[1] + 4*x[4]*x[3]*x[1] - 2*x \\
& [3]^2*x[1] + 2*x[6]*x[2]*x[1] + 2*x[4]*x[2]*x[1] - x[3]*x[2]*x \\
& [1] + x[2]^2*x[1] + 2*x[5]*x[1]^2 - 2*x[4]*x[1]^2 - x[2]*x[1]^2 \\
& + x[6]*x[3] - x[6]*x[2] - x[4]*x[2] + 2*x[3]*x[2] - 2*x[3]*x \\
& [1] + x[2]*x[1],
\end{aligned}$$

```

- x[6]^2*x[4]^2 + x[5]^2*x[4]^2 + x[6]*x[4]^3 + x[5]*x[4]^3 + x
[6]^2*x[4]*x[3] - x[5]^2*x[4]*x[3] - 2*x[6]*x[4]^2*x[3] - 2*x
[5]*x[4]^2*x[3] + x[6]*x[4]*x[3]^2 + x[5]*x[4]*x[3]^2 - 2*x
[5]*x[4]^2*x[2] - x[4]^3*x[2] - x[6]*x[4]*x[3]*x[2] + x[5]*x
[4]*x[3]*x[2] + 2*x[4]^2*x[3]*x[2] + x[6]*x[3]^2*x[2] + x[5]*x
[3]^2*x[2] - x[4]^2*x[2] + x[4]^2*x[2]^2 - x[3]^2*x[2]^2 +
+ 2*x[6]*x[4]^2*x[1] - x[4]^3*x[1] - 2*x[6]*x[4]*x[3]*x[1] +
2*x[4]^2*x[3]*x[1] - x[4]*x[3]^2*x[1] + x[6]*x[4]*x[2]*x[1] +
x[5]*x[4]*x[2]*x[1] - x[6]*x[3]*x[2]*x[1] - x[5]*x[3]*x[2]*x
[1] + x[4]*x[3]*x[2]*x[1] - x[3]^2*x[2]*x[1] - x[4]*x[2]^2*x
[1] + x[3]*x[2]^2*x[1] - x[4]^2*x[1]^2 + x[4]*x[3]*x[1]^2 - x
[4]*x[2]*x[1]^2 + x[3]*x[2]*x[1]^2 + x[6]^3 + x[6]*x[5]^2 - x
[6]^2*x[4] + x[5]*x[4]*x[3] - x[5]*x[3]^2 - 2*x[6]*x[5]*x[2] +
x[6]*x[4]*x[2] - x[5]*x[4]*x[2] - x[4]^2*x[2] + x[3]^2*x[2] +
x[6]*x[2]^2 + x[4]*x[2]^2 + x[3]*x[2]^2 - 2*x[6]^2*x[1] - x
[5]^2*x[1] + x[6]*x[4]*x[1] + 2*x[5]*x[4]*x[1] + 2*x[4]^2*x[1] -
x[5]*x[3]*x[1] - 4*x[4]*x[3]*x[1] + 2*x[3]^2*x[1] + 2*x[5]*x
[2]*x[1] - 3*x[4]*x[2]*x[1] - 2*x[2]^2*x[1] + x[6]*x[1]^2 + x
[4]*x[1]^2 - x[3]*x[1]^2 + x[2]*x[1]^2 + x[5]*x[3] - x[5]*x[2]
+ x[4]*x[2] - 3*x[3]*x[2] + x[2]^2 + x[3]*x[1] - x[6] + x[4]

],
[
x[6]*x[5] - x[5]^2 + x[6] - x[5],
x[6]^2 - x[6]*x[5] - x[6] + x[5] + 1,
- x[6]^2*x[4] + 2*x[6]*x[5]*x[4] - x[5]^2*x[4] + x[6]*x[4]^2 - x
[5]*x[4]^2 - x[6]*x[4]*x[3] + x[5]*x[4]*x[3] - x[6]^2*x[2] +
2*x[6]*x[5]*x[2] - x[5]^2*x[2] - x[6]*x[4]*x[2] + x[5]*x[4]*x
[2] + x[6]*x[4]*x[1] - x[5]*x[4]*x[1] + x[6]*x[3] - x[5]*x[3]
+ x[4]*x[2] - x[2]^2 + x[6]*x[1] - x[5]*x[1] - x[4]*x[1] + x
[2]*x[1],
x[6]^2*x[4] - 2*x[6]*x[5]*x[4] + x[5]^2*x[4] - x[6]*x[4]^2 + x[5]*x
[4]^2 + x[6]*x[4]*x[3] - x[5]*x[4]*x[3] + x[6]*x[2]^2 - x[5]*x
[2]^2 - x[6]*x[4]*x[1] + x[5]*x[4]*x[1] + x[5]*x[4] - x[6]*x[3]
+ x[5]*x[3] - x[4]*x[3] - x[5]*x[2] + x[3]*x[2] + x[3] - x[1],
- x[6]^2*x[4] + 2*x[6]*x[5]*x[4] - x[5]^2*x[4] + x[6]*x[4]^2 - x
[5]*x[4]^2 - x[6]*x[4]*x[3] + x[5]*x[4]*x[3] - x[6]*x[4]*x[2]
+ x[5]*x[4]*x[2] - x[6]*x[3]*x[2] + x[5]*x[3]*x[2] + x[6]*x
[4]*x[1] - x[5]*x[4]*x[1] + x[6]*x[2]*x[1] - x[5]*x[2]*x[1] +
x[6]*x[4] - x[4]^2 + x[6]*x[3] - x[5]*x[3] - x[6]*x[2] + x[4]*x
[2]
]
]);
mat1111 := Matrix(Ground,
[
- x[6]*x[4] + x[5]*x[4] + x[4]^2 + x[6]*x[3] - x[5]*x[3] - 2*x
[4]*x[3] + x[3]^2 - x[4]*x[2] + x[3]*x[2] + x[4]*x[1] - x[3]*x
[1] - x[2] + x[1],

```

```

x[6]^3 - 2*x[6]^2*x[5] + x[6]*x[5]^2 - 2*x[6]^2*x[4] + 2*x[6]*x
[5]*x[4] + x[6]*x[4]^2 + 2*x[6]^2*x[3] - 2*x[6]*x[5]*x[3] - 2*x
[6]*x[4]*x[3] + x[6]*x[3]^2 + 2*x[6]^2*x[2] - 2*x[6]*x[5]*x[2]
- 2*x[6]*x[4]*x[2] + 2*x[6]*x[3]*x[2] + x[6]*x[2]^2 - 3*x[6]^2*
x[1] + 4*x[6]*x[5]*x[1] - x[5]^2*x[1] + 4*x[6]*x[4]*x[1] - 2*x
[5]*x[4]*x[1] - x[4]^2*x[1] - 4*x[6]*x[3]*x[1] + 2*x[5]*x[3]*x
[1] + 2*x[4]*x[3]*x[1] - x[3]^2*x[1] - 4*x[6]*x[2]*x[1] + 2*x
[5]*x[2]*x[1] + 2*x[4]*x[2]*x[1] - 2*x[3]*x[2]*x[1] - x[2]^2*x
[1] + 3*x[6]*x[1]^2 - 2*x[5]*x[1]^2 - 2*x[4]*x[1]^2 + 2*x[3]*x
[1]^2 + 2*x[2]*x[1]^2 - x[1]^3 - x[6]*x[3] + x[5]*x[3] + x[4]*x
[3] - x[3]^2 + x[6]*x[2] - x[5]*x[2] - x[4]*x[2] + x[2]^2 + x
[3]*x[1] - x[2]*x[1] - x[6] + x[4]

],
[
x[6] - x[5] + 1,
- x[6]*x[4] + x[5]*x[4] + x[4]^2 - x[4]*x[3] + x[6]*x[2] - x[5]*x
[2] - 2*x[4]*x[2] + x[3]*x[2] + x[2]^2 + x[4]*x[1] - x[2]*x[1]
+ x[3] - x[1]
]
]);

```

```

Minor4 := Minors(mat4,2);
Minor31 := Minors(mat31,3);
Minor22 := Minors(mat22,3);
Minor211 := Minors(mat211,3);
Minor1111 := Minors(mat1111,2);

LMinor4 := Minors(mat4,1);
LMinor31 := Minors(mat31,2);
LMinor22 := Minors(mat22,2);
LMinor211 := Minors(mat211,2);
LMinor1111 := Minors(mat1111,1);

Upper4 := Ideal(Minor4);
Upper31 := Ideal(Minor31);
Upper22 := Ideal(Minor22);
Upper211 := Ideal(Minor211);
Upper1111 := Ideal(Minor1111);

Lower4 := Ideal(LMinor4);
Lower31 := Ideal(LMinor31);
Lower22 := Ideal(LMinor22);
Lower211 := Ideal(LMinor211);
Lower1111 := Ideal(LMinor1111);

```

Due to the limitations on the size of the output of the Magma online calculator, we then executed the commands

```
RadicalDecomposition(Upper4 + Upper31 + Upper22 + Upper211 +
Upper1111);
```

```

RadicalDecomposition(Lower4 + Upper31 + Upper22 + Upper211 +
Upper1111);
RadicalDecomposition(Upper4 + Lower31 + Upper22 + Upper211 +
Upper1111);
RadicalDecomposition(Upper4 + Upper31 + Lower22 + Upper211 +
Upper1111);
RadicalDecomposition(Upper4 + Upper31 + Upper22 + Lower211 +
Upper1111);
RadicalDecomposition(Upper4 + Upper31 + Upper22 + Upper211 +
Lower1111);

```

one by one, as otherwise the output gets truncated. This produces the results presented in the Appendix of [1].

REFERENCES

- [1] Murray Bremner and Vladimir Dotsenko, *Classification of regular parametrized one - relation operads*, 2015.