

MA1S11 (Dotsenko) Sample questions for the calculus part of 1S11

Michaelmas 2013

1. Compute the limit $\lim_{x \rightarrow 0} \frac{\tan(7x)}{e^{3x}-1}$.
2. From the first principles, prove that the derivative of the function $f(x) = \frac{1}{\sqrt{x}}$ is given by the formula $\frac{-1}{2x\sqrt{x}}$.

3. Is the function

$$f(x) = \begin{cases} x^3 \sin \frac{1}{x}, & x \neq 0, \\ 0, & x = 0 \end{cases}$$

continuous at $x = 0$? differentiable at $x = 0$? twice differentiable at $x = 0$? Explain your answer.

4. Compute the derivatives:

(a) $(\tan(7 + 5 \ln x))^3$; (b) $\cos^{-1} x$; (c) $x^{1/x}$; (d) $\ln\left(\frac{e^x}{1+e^x}\right)$.

5. Compute $f'(\pi/6)$, if $f(x) = \tan^{-1}(\cos x)$.
6. Compute $f'(e)$ for $f(x) = \frac{x^3}{\ln x}$.
7. "The slope of the tangent to the curve $y = ax^3 + bx + 4$ at the point $(2, 14)$ on that curve is 21." Find the values of a and b for which it is true.
8. For $f(x) = \sin(\ln x)$, show that $x^2 f'' + x f' + f = 0$.
9. Determine relative extrema and inflection points of the graph $y = x^3 - 8x^2 + 16x$, and draw a rough sketch of that graph.
10. Show that among all the rectangles of area A , the square has the minimum perimeter.
11. The concentration C of an antibiotic in the bloodstream after time t is given by

$$C = \frac{5t}{1 + \frac{t^2}{k^2}}$$

for a certain constant k . If it is known that the maximal concentration is reached at $t = 6$ hours, find the value of k .

12. Evaluate the integrals

(a) $\int \frac{\sin 2\theta}{1+\cos 2\theta} d\theta$; (b) $\int \frac{x dx}{1+x^2}$; (c) $\int x^3 \sqrt[3]{1-4x} dx$.

13. Evaluate the integrals

(a) $\int_{1/2}^1 \frac{3}{2x} dx$; (b) $\int_0^\pi \frac{\cos^2 x}{1+\sin x} dx$; (c) $\int_{e^{-1}}^e \frac{\sqrt{1-(\ln x)^2}}{x} dx$.

14. Find a positive value of k for which the area under the graph of $y = e^{3x}$ over the interval $[0, k]$ is 11 square units.
15. Compute the area of the region between the graphs $y = xe^x$ and $y = x^2 e^x$.