

MA1S11 (Dotsenko) Tutorial/Exercise Sheet 1

Week 2, Michaelmas 2013

Please hand in your work in the end of the tutorial. Make sure you put your name and student ID number on what you hand in.

A complete solution to every question is worth 2 marks.

Reminder:

- The *domain* of a function f consists of all values of x for which $f(x)$ is defined. The *range* of f consists of all values $f(x)$ when x is varied over the domain of f . If no domain is given explicitly, the *natural domain* of a function f given by an algebraic expression is the set of all values of x for which $f(x)$ is defined and real.
- Given two functions f and g we define their *composition* $f \circ g$ by

$$(f \circ g)(x) = f(g(x)),$$

so x must be in the domain of g and $g(x)$ in the domain of f for this to make sense. The composition of functions can be iterated, so if there is a third function h we may define

$$(f \circ g \circ h)(x) = f(g(h(x))),$$

and so on.

Questions

1. Using the factorisation $x^2 - x - 6 = (x+2)(x-3)$, find the natural domain of $\sqrt{x^2 - x - 6}$.
2. Explain why the domain of $\sqrt{x+2}\sqrt{x-3}$ is different from that of $\sqrt{x^2 - x - 6}$.
3. Plot the graph of the function

$$\text{sign}(x) := \frac{x}{|x|},$$

and determine the natural domain and the range of this function.

4. Plot the graphs of $\text{sign}(x+1)$ and of $\text{sign}(-x)$.
5. What is the domain of $f \circ g \circ h$, if $f(x) = 1 - x$, $g(x) = \frac{1}{x}$, and $h(x) = x^2 + 1$?