MA1S11 (Dotsenko) Tutorial/Exercise Sheet 10

Week 12, Michaelmas 2013

Please hand in your work in the end of the tutorial. Make sure you put your name and student ID number on what you hand in.

A complete solution to each question is worth 2 marks.

Reminder:

- The **antiderivative** and the **indefinite integral**: for a function $f(x)$, the function $F(x)$ is its *anti-derivative* if
$$\frac{dF(x)}{dx} = f(x)$$
The *indefinite integral* is the family of all anti-derivatives
$$\int f(x)\,dx = F(x) + C$$
where C is *the arbitrary constant of integration.*

- **Integration table**

$f(x)$	$\int f(x)\,dx$
x^r $(r \neq -1)$	$\frac{x^{r+1}}{r+1} + C$
e^x	$e^x + C$
$1/x$	$\ln x + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\cos^2 x$	$\tan x + C$

- **Linearity**: if
$$\int f(x)\,dx = F(x) + C, \qquad \int g(x)\,dx = G(x) + C$$
then
$$\int \lambda f(x)\,dx = \lambda F(x) + C \qquad \int [f(x) + g(x)]\,dx = F(x) + G(x) + C$$
where λ is a constant.

- The **Fundamental Theorem of Calculus**: we have
$$\int_a^b F'(x)dx = F(b) - F(a)$$
and
$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

- **u-substitution**:

$$\int f(g(x))g'(x)dx = \int f(u)du$$

where $u = g(x)$; another way of saying this is that inside the integral

$$\frac{du}{dx}dx = du.$$

For the definite integral replace the x limits with u limits

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du.$$

- **Integration by parts**: If $F'(x) = f(x)$, and $G'(x) = g(x)$, then the product rule from the point of view of antiderivatives can be written in the form:

$$\int G(x)f(x)\,dx = F(x)G(x) - \int F(x)g(x)\,dx,$$

or equivalently using the notation $du = u'(x)\,dx$ from above

$$\int G\,dF = FG - \int F\,dG.$$

Questions

1. Evaluate the indefinite integrals

$$a) \int \frac{(2+3\sqrt{x})^{20}}{\sqrt{x}}\,dx \qquad b) \int x^3\sqrt{1+4x}\,dx$$

2. Evaluate the integrals

$$a) \int_0^{\pi/2} \sin x\,dx \qquad b) \int_1^2 \left(y^2 - y^{-3}\right) dy \qquad \text{and} \qquad \int_2^1 \left(y^2 - y^{-3}\right) dy$$

3. Evaluate the integrals

$$a) \int_{-1}^{5} (3+2w)(3w+w^2)^5\,dw \qquad b) \int_{-\pi}^{\pi/2} \cos x e^{\sin x}\,dx$$

4. Evaluate the integral

$$\int_{-1}^{2} \sqrt{2+|x|}\,dx.$$

5. Show that

$$\int_{1/2}^{2} \frac{1}{x}\sin\left(x - \frac{1}{x}\right)\,dx = 0$$

(*Hint*: try the u-substitution $u = 1/x$.)