MA1S11 (Dotsenko) Tutorial/Exercise Sheet 5

Week 6, Michaelmas 2013

Please hand in your work in the end of the tutorial. Make sure you put your name and student ID number on what you hand in.

A complete solution to every question is worth 2 marks.

Reminder:

1. Given a function f, the function f' defined by the formula

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

is called the *derivative of f with respect to x*. The domain of f' consists of all x for which the limit exists.

2. To determine the equation of the tangent line to the graph of f at $x = x_0$ when we can compute the derivative function:

- Evaluate $f(x_0)$; the point of tangency is $(x_0, f(x_0))$.
- Evaluate $f'(x_0)$; that is the slope of the tangent line.
- Use the point of tangency and the slope in the point–slope equation of the line to get

$$y - f(x_0) = f'(x_0)(x - x_0),$$

or equivalently

$$y = f'(x_0)(x - x_0) + f(x_0).$$

3. Every polynomial

$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

is differentiable everywhere, and

$$f'(x) = a_1 + 2a_2 x + 3a_3 x^2 + \cdots + n a_n x^{n-1}.$$

Questions

1. Let $f(x) = x^{5/3}$ and $g(x) = \sin x$. Compute $f'(0)$ and $g'(0)$.
2. Find the equation for the tangent line to the graph $y = \sqrt{x}$ at $x = 4$.

3. Is the function $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, \ x \neq 0, \\ 0, \quad x = 0. \end{cases}$ differentiable at the point $x = 0$?

4. Find the values of x_0 for which the tangent line to the graph $y = x^3 - x$ at $x = x_0$ is parallel to the line $y = x$, and write down equations for the corresponding tangent lines.

5. A person drops a coin from the roof of a skyscraper which is 218 metres above the street level. The position of the coin (in metres above water) as a function of time (in seconds) is given by

$$s(t) = 218 - \frac{1}{2}gt^2,$$

where $g = 9.81m/s^2$ (metres per seconds squared) is the acceleration due to the gravitational force.

   - How long will it take for the coin to reach the street level?
   - What is the *instantaneous* velocity of the coin as a function of t?
   - What is the instantaneous velocity of the coin at the end of the fall?