MA1S11 (Dotsenko) Tutorial/Exercise Sheet 6

Week 8, Michaelmas 2013

Please hand in your work in the end of the tutorial. Make sure you put your name and student ID number on what you hand in.

A complete solution to every question is worth 2 marks.

Reminder:

1. The derivative function is defined as

$$f'(x) = \frac{d}{dx} f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}. \tag{1}$$

2. If $f(x) = x^n$ then $f'(x) = nx^{n-1}$. This holds for any n, integer, fractional, etc.
3. For any function f and any real number c

$$(cf)' = cf', \qquad (f+g)' = f' + g'. \tag{2}$$

4. Derivatives of products and quotients:

$$(fg)' = fg' + f'g, \qquad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}. \tag{3}$$

5. Derivatives of trigonometric functions:

$$\frac{d}{dx} \cos x = -\sin x, \qquad \frac{d}{dx} \sin x = \cos x \tag{4}$$

6. Chain rule: for $u = g(x)$

$$\frac{d}{dx} f(g(x)) = \frac{df}{du} \frac{du}{dx} = f'(u) g'(x) = f'(g(x)) g'(x). \tag{5}$$

Questions

1. Differentiate

$$f(x) = \frac{1}{x^2} + 4x^{3/2}, \tag{6}$$

and

$$f(x) = x + \frac{1}{x} + \frac{3}{4} x^{4/3}. \tag{7}$$

2. Show that for the curves $y = 1/x$ and $y = 1/(2-x)$, their tangent lines at the point where those two curves meet are perpendicular to one another.

3. Using the chain rule and rules for derivatives of trigonometric functions, compute the derivative function of $f(x) = \sin^2 x + \cos^2 x$. Explain why your answer is consistent with the trigonometric identity $\sin^2 x + \cos^2 x = 1$.

4. Differentiate
$$f(x) = x \cos x + \sqrt{1 - x^2}, \tag{8}$$
and
$$f(x) = \frac{\sin x}{x + 3\sqrt[3]{x}}. \tag{9}$$

5. Let $f(x) = \sin x$, with the domain $(-\pi/2, \pi/2)$, where this function is increasing and therefore is invertible. Apply the chain rule to the equation
$$f\left(f^{-1}(x)\right) = x \tag{10}$$
to show that the derivative of $f^{-1}(x)$ is $\frac{1}{\sqrt{1-x^2}}$. (*Hint*: you may need the identity $\cos^2 x + \sin^2 x = 1$).