UNIVERSITY OF DUBLIN

XMA1111

TRINITY COLLEGE

Faculty of Science

school of mathematics

JF Mathematics
JF Theoretical Physics
JF Two Subject Mod

Trinity Term 2011

Course 1111

Dr. Vladimir Dotsenko

For each task, the number of points you can get for a complete solution of that task is printed next to it.

You may use all statements proved in class and in home assignments; when using some statement, you should formulate it clearly, e.g. "in class, we proved that if A is invertible, then the reduced row echelon form of A is the identity matrix".

All vector spaces unless otherwise specified are over complex numbers.

Non-programmable calculators are permitted for this examination.

1. (25 points) Using elementary row operations, compute the inverse of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{pmatrix}$.

   Find a polynomial $f(t)$ of degree at most 2 for which $f(1) = 1$, $f(3) = 0$, $f(4) = 11$.

2. (25 points) Write down the definition of an even permutation. For each of the permutations $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 3 & 2 & 7 & 6 & 5 & 1 \end{pmatrix}$, $\begin{pmatrix} 4 & 8 & 2 & 3 & 7 & 6 & 5 & 1 \\ 5 & 2 & 8 & 4 & 6 & 1 & 3 & 7 \end{pmatrix}$, and $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 8 & 2 & 5 & 6 & 7 & 3 & 1 \end{pmatrix}$ determine whether it is odd or even:

3. (a) (10 points) Prove that for two square matrices A and B of the same size we always have $\mathrm{tr}(AB) = \mathrm{tr}(BA)$.

   (b) (15 points) How many *distinct* numbers can there be among the six traces

   $$\mathrm{tr}(ABC), \mathrm{tr}(ACB), \mathrm{tr}(BCA), \mathrm{tr}(BAC), \mathrm{tr}(CBA), \mathrm{tr}(CAB)?$$

   for different choices of square matrices A, B, C of the same size? For each variant of the answer, give an example.

4. (25 points) For the matrix

$$A = \begin{pmatrix} 1 & 1 & 2 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 2 \\ 2 & 1 & 1 & 1 & 1 & 2 & 5 \\ -1 & 2 & 1 & 1 & 0 & 1 & 1 \end{pmatrix},$$

compute the dimension and find a basis of the solution space to the system of equations $Ax = 0$.