UNIVERSITY OF DUBLIN
TRINITY COLLEGE

Faculty of Science

school of mathematics

JF Mathematics **Trinity Term 2011**
JF Theoretical Physics
JF Two Subject Mod

Course 1111, a sample exam paper

Dr. Vladimir Dotsenko

For each task, the number of points you can get for a complete solution of that task is printed next to it.

You may use all statements proved in class and in home assignments; when using some statement, you should formulate it clearly, e.g. "in class, we proved that if A is invertible, then the reduced row echelon form of A is the identity matrix".

All vector spaces unless otherwise specified are over complex numbers.

Non-programmable calculators are permitted for this examination.

1. (25 points) Denote by A the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$ and by b the vector $\begin{pmatrix} 5 \\ -1 \\ 2 \end{pmatrix}$. List all minors and all cofactors of A, and write down the expansion of $\det(A)$ along the second row and along the third column. Show how to use the Cramer's rule to solve the system $Ax = b$.

2. (25 points) Describe all possible values of i, j, k and l for which the term

$$a_{4k}a_{35}a_{il}a_{67}a_{j1}a_{23}a_{14}$$

occurs in the expansion of a 7×7 determinant with coefficient -1.

3. (25 points) Compute the determinant of the $n \times n$-matrix $\begin{pmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & 0 & 1 & \dots & 1 \\ \vdots & \dots & \ddots & \dots & \vdots \\ \vdots & \dots & \dots & \ddots & \vdots \\ 1 & 1 & \dots & 1 & 0 \end{pmatrix}$ (all diagonal entries are equal to 0, all off-diagonal entries are equal to 1).

4. (25 points) A 3×3-checkboard whose cells are filled in with 9 real numbers is called a magic square if all its row sums are pairwise equal, and equal to all of its column sums. Prove that the set of all magic squares forms a subspace of $\mathbb{R}^9$, compute the dimension of this space, and find a basis of this space.