UNIVERSITY OF DUBLIN
TRINITY COLLEGE

Faculty of Science

school of mathematics

JF Mathematics **Midterm Test 2011**
JF Theoretical Physics
SF Two Subject Mod

MA1212: Linear Algebra

March 8, 2011 CLLT 17.15–18.45

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This test should be completed in 1.5 hours

For each task, the number of points you can get for a complete solution of that task is printed next to it.

All vector spaces unless otherwise specified are over complex numbers.

You may use all statements proved in class and in home assignments; when using some statement, you should formulate it clearly, e.g. "in class, we proved that if A is invertible, then the reduced row echelon form of A is the identity matrix".

Non-programmable calculators are permitted.

1. (30 points) Define the rank of a linear operator. Compute the rank of the linear operator $A\colon \mathbb{R}^8 \to \mathbb{R}^4$ whose matrix relative to the standard bases is

$$\begin{pmatrix} 3 & 5 & 4 & 2 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 2 & 4 & 1 & 1 \\ 4 & 1 & 1 & 0 & 1 & 0 & 2 & -1 \\ -1 & 0 & 1 & 1 & 2 & 1 & 1 & -2 \end{pmatrix}$$

2. In the vector space $V = \mathbb{R}^5$, consider the subspace U spanned by the vectors

$$\begin{pmatrix} 2 \\ 2 \\ 1 \\ 7 \\ -3 \end{pmatrix}, \quad \begin{pmatrix} -4 \\ 1 \\ -12 \\ 6 \\ -4 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 1 \\ 3 \\ 4 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ 3 \\ 1 \\ 2 \end{pmatrix}, \text{ and } \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}.$$

   (a) (15 points) Compute $\dim U$.

   (b) (15 points) Which of the vectors $\begin{pmatrix} 4 \\ 0 \\ 5 \\ -3 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 8 \\ 4 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \\ 4 \\ 0 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} 1 \\ 0 \\ 5 \\ 0 \\ 2 \end{pmatrix}$ belong to U?

3. (40 points) Consider the matrices

$$A = \begin{pmatrix} -3 & 1 & 0 \\ -1 & -1 & 0 \\ -1 & -2 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Describe the Jordan normal form and find some Jordan basis for A. Do A and B represent the same linear operator in different coordinate systems? Explain your answer.