UNIVERSITY OF DUBLIN
TRINITY COLLEGE

Faculty of Science

school of mathematics

JF Mathematics **Sample Midterm Test 2011**
JF Theoretical Physics
SF Two Subject Mod

MA1212: Linear Algebra

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This test should be completed in 1.5 hours

For each task, the number of points you can get for a complete solution of that task is printed next to it.

All vector spaces unless otherwise specified are over complex numbers.

You may use all statements proved in class and in home assignments; when using some statement, you should formulate it clearly, e.g. "in class, we proved that if A is invertible, then the reduced row echelon form of A is the identity matrix".

Non-programmable calculators are permitted.

1. (10 points) Which of the mappings $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$ from the vector space of all polynomials in one variable to the same space are linear operators? Explain your answers.

$$(\mathcal{A}p)(t) = p(t+1) - p(t),$$
$$(\mathcal{B}p)(t) = p(t)p'(t),$$
$$(\mathcal{C}p)(t) = p(t+1) + p'(t),$$
$$(\mathcal{D}p)(t) = p(t+1) - 1.$$

2. (15 points) Under what condition a subspace U of a vector space V is said to be an invariant subspace of a linear operator $A\colon V \to V$? Is the subspace U of $\mathbb{R}^4$ spanned by $\begin{pmatrix} 1 \\ 1 \\ 4 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ -1 \\ -1 \\ 1 \end{pmatrix}$ an invariant subspace of the operator A whose matrix relative to the standard basis is

$$\begin{pmatrix} 0 & 3 & -3 & -1 \\ 1 & 3 & -1 & 0 \\ 7 & 12 & 2 & 3 \\ -3 & -6 & 0 & -1 \end{pmatrix}?$$

Explain your answer.

3. (25 points) Determine the Jordan normal form and find some Jordan basis for the matrix $A = \begin{pmatrix} 3 & -3 & 1 \\ 2 & -2 & 1 \\ 2 & -3 & 2 \end{pmatrix}$. Determine if A and the matrix $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ represent the same linear operator relative to different bases.