MA 1212: Linear Algebra II
Tutorial problems, January 29, 2015

1. For the space $\mathbb{R}^3$ with the standard inner product, find the orthogonal basis e_1, e_2, e_3 obtained by Gram–Schmidt orthogonalisation from $f_1 = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}$, $f_2 = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$, $f_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

2. Show that the formula

$$\left(\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}\right) = x_1x_2 + \frac{1}{2}(x_1y_2 + x_2y_1) + y_1y_2.$$

defines a scalar product on $\mathbb{R}^2$, and find an orthonormal basis of $\mathbb{R}^2$ with respect to that scalar product.

3. For the vector space of all polynomials in t of degree at most 3 and the scalar product on this space given by

$$(p(t), q(t)) = \int_{-1}^{1} p(t)q(t)\,dt,$$

find the result of Gram–Schmidt orthogonalisation of the vectors $1, t, t^2, t^3$.