MA 1212: Linear Algebra II
Tutorial problems, February 12, 2015

1. For the matrix $\begin{pmatrix} 3 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix}$ of a certain bilinear form, compute the determinants $\Delta_1, \Delta_2, \Delta_3$, and determine the signature of the corresponding quadratic form.

2. Use the Sylvester's criterion to find all values of the parameter a for which the quadratic form $(18+a)x_1^2+3x_2^2+ax_3^2+10x_1x_2-(8+2a)x_1x_3-4x_2x_3$ on $\mathbb{R}^3$ is positive definite.

3. Compute the eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, and determine the signature of the quadratic form

$$q(x_1e_1 + x_2e_2 + x_3e_3) = x_1x_2 + x_2x_3.$$

4. Let

$$\varphi(x_1, x_2) = \sin^2(x_1 - x_2) - e^{\frac{1}{2}(x_1^2+x_2^2)+2cx_1x_2}.$$

Furthermore, let A be the symmetric 2×2-matrix with entries $a_{ij} = \frac{\partial^2\varphi}{\partial x_i \partial x_j}(0, 0, 0)$.

(**a**) Write down the matrix A.

(**b**) Determine all values of the parameter c for which the corresponding quadratic form is positive definite.

(**c**) Does φ have a local minimum at the origin $(0, 0)$ for $c = -3/5$?