MA 1111: Linear Algebra I
Homework problems due November 5, 2015

Solutions to this problem sheet are to be handed in after our class at 3pm on Thursday. Please attach a cover sheet with a declaration `http://tcd-ie.libguides.com/plagiarism/declaration` confirming that you know and understand College rules on plagiarism.

Please put your name and student number on each of the sheets you are handing in. (Or, ideally, staple them together).

1. (**a**) Without directly evaluating the determinant, explain why

$$\det\begin{pmatrix} x^2 & 2 & 0 \\ x & 1 & 0 \\ 2 & 1 & -5 \end{pmatrix} = 0$$

for $x = 0$ and $x = 2$.

(**b**) Show that for each choice of α, β, and γ the matrix

$$\begin{pmatrix} \cos^2\alpha & \cos^2\beta & \cos^2\gamma \\ \sin^2\alpha & \sin^2\beta & \sin^2\gamma \\ 1 & 1 & 1 \end{pmatrix}$$

is not invertible.

2. For the matrix

$$A = \begin{pmatrix} 2 & 4 & 1 \\ 1 & 3 & 3 \\ 3 & 1 & 5 \end{pmatrix},$$

compute (**a**) all its minors; (**b**) all its cofactors.

3. For the matrix A from Problem 2, evaluate $\det(A)$ using the cofactor expansion along (**a**) the first row; (**b**) the first column; (**c**) the second row; (**d**) the second column; (**e**) the third row; (**f**) the third column; (**g**) write down the adjugate matrix $\mathrm{adj}(A)$ and use it to compute A^{-1}; (**h**) use the Cramer's rule to solve the system $Ax = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

4. (**a**) Suppose that A is an invertible $n \times n$-matrix. Show that $\det(A^{-1}) = \frac{1}{\det(A)}$.

(**b**) Let A and B be square matrices. Assuming that A is invertible, show that $\det(A^{-1}BA) = \det(B)$.

5. (**a**) Prove that if for a $n \times n$-matrix A (with real entries) we have $A^2 = -I$, then n is even.

(**b**) A square matrix A is said to be skew-symmetric, if $A^T = -A$. Show that if A is a skew-symmetric $(2k+1) \times (2k+1)$-matrix, then $\det(A) = 0$.