MA 1111: Linear Algebra I
Tutorial problems, October 7, 2015

1. Let $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ be some vectors in 3d space. Which of the following products are defined (and why):

$$\mathbf{u}\times(\mathbf{v}\times\mathbf{w}),\ \mathbf{v}\times(\mathbf{u}\cdot\mathbf{w}),\ \mathbf{u}\times\mathbf{v}\times\mathbf{w},\ (\mathbf{u}\cdot\mathbf{w})\cdot\mathbf{v},\ \mathbf{u}\cdot(\mathbf{w}\cdot\mathbf{v}),\ (\mathbf{u}\times\mathbf{v})\cdot\mathbf{w},\ \mathbf{u}\cdot\mathbf{v}\cdot\mathbf{w}.$$

2. Show that for all 3d vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ we have

$$\mathbf{u}\times(\mathbf{v}\times\mathbf{w})+\mathbf{v}\times(\mathbf{w}\times\mathbf{u})+\mathbf{w}\times(\mathbf{u}\times\mathbf{v})=0$$

(*Hint*: in class, we proved that $\mathbf{u}\times(\mathbf{v}\times\mathbf{w})=(\mathbf{u}\cdot\mathbf{w})\cdot\mathbf{v}-(\mathbf{u}\cdot\mathbf{v})\cdot\mathbf{w}$.)

3. How many solutions, depending on the parameter a, does the following system of equations have?

$$\begin{cases} x+ay=1,\\ ax+y=0. \end{cases}$$

4. Draw the image of the letter R from the picture under the transformation (**a**) $(x,y)\mapsto(x+2,y+3)$ (all first coordinates of points are increased by 2, all second coordinates — by 3); (**b**) $\dots(x,y)\mapsto(-x,y)$; (**c**) $\dots(x,y)\mapsto(x,2-y)$; (**d**) $\dots(x,y)\mapsto(y,x)$; (**e**) $\dots(x,y)\mapsto(2x,2y)$; (**f**) $\dots(x,y)\mapsto(x,2y)$; (**g**) $\dots(x,y)\mapsto(x,y+x)$. (**h**) What transforsmation one should apply to get the (Russian) letter Я (on the same place)?