

MA 1111: Linear Algebra I
Homework problems for October 12, 2018

Solutions to this problem sheet are to be handed in after our class at 1pm on Friday. Please attach a cover sheet with a declaration

<http://tcd-ie.libguides.com/plagiarism/declaration>

confirming that you know and understand College rules on plagiarism. On the same cover sheet, please put your name, student number, and name of the degree (Maths/TP/TSM), and staple all the sheets together. (Failure to do that may result in misplaced/lost sheets, for which no responsibility can be taken by instructors.)

1. For the following permutations determine whether they are odd or even:

(a) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 3 & 2 & 7 & 6 & 5 & 1 \end{pmatrix}$; (b) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 8 & 2 & 3 & 7 & 6 & 5 & 1 \end{pmatrix}$; (c) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 8 & 2 & 5 & 6 & 7 & 3 & 1 \end{pmatrix}$.

2. List all i, j, k, l for which the permutation $\begin{pmatrix} 5 & 2 & k & 3 & 6 & 1 \\ l & 1 & 3 & i & 6 & j \end{pmatrix}$ is even.

3. Compute the determinant of the matrix (a) $\begin{pmatrix} 1 & 0 & -2 \\ 1 & 1 & 3 \\ 4 & 3 & 1 \end{pmatrix}$; (b) $\begin{pmatrix} 1 & 1 & -2 & -1 \\ 2 & 0 & 3 & -1 \\ 4 & 2 & 3 & 1 \\ 3 & 0 & 0 & 1 \end{pmatrix}$.

4. Compute the determinant (a) of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$; (b) of the matrix $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix}$;

(c) of the $n \times n$ matrix A for which $a_{ij} = \min(i, j)$. (The entry in row i and column j is equal to the minimum of i and j , like in the two previous questions for $n = 3, 4$.)

5. For which values of c does A fail to be invertible:

(a) $A = \begin{pmatrix} 2-c & -1 \\ -1 & 2-c \end{pmatrix}$; (b) $A = \begin{pmatrix} 2 & c-1 & 1 \\ 1+c & 2 & 3 \\ 3 & 4c & -1 \end{pmatrix}$.