MA 1111: Linear Algebra I
Homework problems for October 19, 2018

Solutions to this problem sheet are to be handed in after our class at 1pm on Friday. Please attach a cover sheet with a declaration

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confirming that you know and understand College rules on plagiarism. On the same cover sheet, please put your name, student number, and name of the degree (Maths/TP/TSM), and staple all the sheets together. (Failure to do that may result in misplaced/lost sheets, for which no responsibility can be taken by instructors.)

1. For the matrix

$$A = \begin{pmatrix} 2 & 4 & 1 \\ 1 & 3 & 3 \\ 3 & 1 & 5 \end{pmatrix},$$

compute (**a**) all its minors and (**b**) all its cofactors; evaluate $\det(A)$ using the cofactor expansion along (**c**) the first row, (**d**) the first column, (**e**) the second row, (**f**) the second column, (**g**) the third row, (**h**) the third column.

2. For the matrix from the previous question, (**a**) write down the adjugate matrix and use it to compute A^{-1}; (**b**) use the Cramer's formula to solve the system $Ax = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

3. Assume that for two square matrices A and B of the same size the matrix AB is invertible. (**a**) Show that B is invertible. (*Hint*: if B is not invertible, the system $Bx = 0$ has a nontrivial solution.) (**b**) Show that A is invertible.

4. Let M and N be square matrices, and suppose that M is invertible.
(**a**) Show that $\mathrm{tr}(M^{-1}NM) = \mathrm{tr}(N)$. (*Hint:* use the property $\mathrm{tr}(AB) = \mathrm{tr}(BA)$.)
(**b**) Show that $\det(M^{-1}NM) = \det(N)$.

5. Prove that for every 2×2-matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ we have

$$A^2 - \mathrm{tr}(A) \cdot A + \det(A) \cdot I_2 = 0.$$