MA 1111: Linear Algebra I
Tutorial problems, September 20, 2018

1. $(-2,0)$, $(0,-2)$, or $(2,4)$. In general, if $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are given points, then the fourth point is one of $\mathbf{a}+\mathbf{b}-\mathbf{c}$, $\mathbf{b}+\mathbf{c}-\mathbf{a}$, and $\mathbf{c}+\mathbf{a}-\mathbf{b}$. One of possible ideas is to use the parallelogram rule carefully. Another idea: the midpoint of the segment connecting (a_1,a_2) to (b_1,b_2) has coordinates $(\frac{a_1+b_1}{2},\frac{a_2+b_2}{2})$; use the fact that the center of a parallelogram is the midpoint of each of its diagonals.

2. We have $|\mathbf{a}|=\sqrt{9+25}=\sqrt{34}$, $|\mathbf{b}|=\sqrt{4+1}=\sqrt{5}$, and $\mathbf{a}\cdot\mathbf{b}=6+5=11$. Therefor, $\cos\varphi=\frac{11}{\sqrt{34\cdot 5}}=\frac{11}{\sqrt{170}}$, and $\varphi=\cos^{-1}\left(\frac{11}{\sqrt{170}}\right)$.

3. (**a**) This area, as we know, is equal to the length of the vector product of these vectors. We have $\mathbf{u}\times\mathbf{v}=(4,-1,-1)$, so the area is $\sqrt{4^2+1+1}=\sqrt{18}=3\sqrt{2}$. (**b**) This area is the absolute value of $\mathbf{w}\cdot(\mathbf{u}\times\mathbf{v})$, that is $|12-1|=11$.

4. Since $\mathbf{v}\cdot\mathbf{w}$ is a scalar, the vector product of that expression with a vector is not defined.

5. (**a**) Let us choose one of the vertices of that cube. There are three other vertices connected to it with an edge, at the distance 1, three others which are other endpoints of diagonals of square faces of the cube, at the distance $\sqrt{2}$, and finally, one vertex which is the opposite vertex of the cube; the distance to that vertex is equal to $\sqrt{(\sqrt{2})^2+1^2}=\sqrt{3}$ by Pythagoras' Theorem. (**b**) Note that ABC is an equilateral triangle with side $\sqrt{2}$, so all its angles are $60°$. Alternatively: if we put the vertex A at the origin of a coordinate system, and direct the edges along the sides of the cube, we are asked to find the angles between the vectors $(1,1,0)$ and $(1,0,1)$. The scalar product of those vectors is 1, so $\cos\varphi=\frac{1}{\sqrt{2}\cdot\sqrt{2}}=\frac{1}{2}$, and $\varphi=60°$.