MA 1111: Linear Algebra I
Tutorial problems, October 19, 2018

1. The reduced row echelon form of the matrix $\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$ is the identity matrix, so the vectors form a basis.

2. There are more than two vectors, so they are not linearly independent, and do not form a basis. The first and the third form a basis, hence span $\mathbb{R}^2$, hence the three also span $\mathbb{R}^2$.

3. There are more than three vectors, so they are not linearly independent, and do not form a basis. The matrix formed by the first, the second, and the last vector has the determinant 9, so is invertible, so its reduced row echelon form is the identity matrix, so those three vectors span $\mathbb{R}^3$, hence all the five vectors span $\mathbb{R}^3$.

4. The matrix formed by these vectors has the determinant 2, so is invertible, so its reduced row echelon form is the identity matrix, so those three vectors form a basis.

5. (**a**) Suppose that $c_1(u-2w)+c_2(v+w)+c_3w=0$. Since

$$c_1(u-2w)+c_2(v+w)+c_3w=c_1u+c_2v+(-2c_1+c_2+c_3)w,$$

and u, v, w are linearly independent, we have $c_1=c_2=-2c_1+c_2+c_3=0$, from which we deduce $c_1=c_2=c_3=0$.

(**b**) Suppose that we want to find coefficients c_1, c_2, c_3 that give

$$c_1(u-2w)+c_2(v+w)+c_3w=b.$$

Since

$$c_1(u-2w)+c_2(v+w)+c_3w=c_1u+c_2v+(-2c_1+c_2+c_3)w,$$

we may instead solve $c_1u+c_2v+(-2c_1+c_2+c_3)w=b$. By assumption, the system of vectors u, v, w is complete, so we can write $a_1u+a_2v+a_3w=b$, and it is enough to set $c_1=a_1$, $c_2=a_2$, and $c_3=2c_1-c_2+a_3$.