

MA 1112: Linear Algebra II  
Homework problems for February 4, 2019

Solutions to this problem sheet are to be handed in after our class at 11am on Monday. Please attach a cover sheet with a declaration <http://tcd-ie.libguides.com/plagiarism/declaration> confirming that you know and understand College rules on plagiarism. On the same cover sheet, please put your name, student number, and name of the degree (Maths/TP/TSM), and staple all the sheets together. (Failure to do that may result in misplaced/lost sheets, for which no responsibility can be taken by instructors.)

We consider the two following subspaces of  $\mathbb{R}^4$ : the subspace  $\mathbf{U}_1$  spanned by the vectors

$$\begin{pmatrix} 0 \\ -1 \\ -6 \\ -9 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -5 \\ -7 \end{pmatrix}, \begin{pmatrix} 6 \\ 5 \\ 0 \\ 3 \end{pmatrix},$$

and the subspace  $\mathbf{U}_2$  spanned by the vectors

$$\begin{pmatrix} 5 \\ 6 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 6 \\ 6 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 7 \\ 1 \\ -15 \\ -7 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 2 \\ 2 \end{pmatrix},$$

1. Find a basis of  $\mathbf{U}_1$  and a basis of  $\mathbf{U}_2$ .
2. Find a basis for the intersection  $\mathbf{U}_1 \cap \mathbf{U}_2$ .
3. Compute a basis of  $\mathbf{U}_1$  relative to  $\mathbf{U}_1 \cap \mathbf{U}_2$ .
4. Compute a basis of  $\mathbf{U}_2$  relative to  $\mathbf{U}_1 \cap \mathbf{U}_2$ .

5. Is the subspace spanned by the vectors  $\mathbf{v}_1 = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$  and  $\mathbf{v}_2 = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$  an invariant subspace of

the linear transformation  $\varphi$  of  $\mathbb{R}^3$  that multiplies every vector by the matrix  $\mathbf{A} = \begin{pmatrix} -1 & -9 & -6 \\ 6 & 20 & 12 \\ -9 & -24 & -13 \end{pmatrix}$ ?

Explain your answer.