MA 1112: Linear Algebra II
Homework problems for March 11, 2019

You do not have any homework to hand in on February 25: your only homework till then is preparing for the midterm test.

Solutions to this problem sheet are to be handed in after our class at 11am on Monday March 11. Please attach a cover sheet with a declaration `http://tcd-ie.libguides.com/plagiarism/declaration` confirming that you know and understand College rules on plagiarism. On the same cover sheet, please put your name, student number, and name of the degree (Maths/TP/TSM), and staple all the sheets together. (Failure to do that may result in misplaced/lost sheets, for which no responsibility can be taken by instructors.)

1. Which of the following matrices represent the same linear transformations relative to different bases? Explain your answer. (*Hint*: two matrices represent the same linear transformations relative to different bases if their Jordan normal forms are the same; note that you only need to determine the Jordan normal form (sizes of blocks for various eigenvalues), and not a Jordan basis).

$$\begin{pmatrix} -3 & -1 & 0 \\ -1 & -1 & 0 \\ -1 & -2 & 1 \end{pmatrix}, \begin{pmatrix} 9 & 5 & 2 \\ -16 & -9 & -4 \\ 2 & 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}, \begin{pmatrix} -2 & -4 & 16 \\ 0 & 2 & 0 \\ -1 & -1 & 6 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

2. (**a**) Find all eigenvalues and eigenvectors of the matrix

$$B = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}.$$

(**b**) Find the Jordan normal form of the matrix B, and a matrix C which is the transition matrix to some Jordan basis of B.

(**c**) Find a closed formula for B^n, and use it to find a closed formula for the n^{th} terms of the sequences $\{x_m\}$, $\{y_m\}$ defined recursively as follows:

$$x_0 = 1, y_0 = -5,$$
$$x_{k+1} = x_k - y_k, \quad y_{k+1} = x_k + 3y_k.$$

3. The sequence $a_1, a_2, \ldots$ is defined recursively: $a_{n+2} = 14a_{n+1} - 49a_n$, $a_0 = -1$, $a_1 = 1$. Find an explicit formula for a_n.

4. Let

$$A_n = \begin{pmatrix} 5 & 2 & 0 & 0 & \ldots & 0 \\ 2 & 5 & 2 & 0 & \ldots & 0 \\ 0 & 2 & 5 & 2 & \ldots & 0 \\ \vdots & \vdots & \ldots & \ddots & \ldots & \vdots \\ 0 & 0 & \ldots & 2 & 5 & 2 \\ 0 & 0 & \ldots & \ldots & 2 & 5 \end{pmatrix}$$

be the $n \times n$-matrix for which all diagonal elements are equal to 5, all elements on the diagonals next to the main are equal to 2, all other elements are equal to 0.

(**a**) Show that $\det(A_n) = 5\det(A_{n-1}) - 4\det(A_{n-2})$ for all $n \geqslant 3$.

(**b**) Use the recursive formula you obtained to find a closed formula for $\det(A_n)$.