MA 1112: Linear Algebra II
Homework problems for April 1, 2019

Solutions to this problem sheet are to be handed in after our class at 11am on Monday April 1. Please attach a cover sheet with a declaration `http://tcd-ie.libguides.com/plagiarism/declaration` confirming that you know and understand College rules on plagiarism. On the same cover sheet, please put your name, student number, and name of the degree (Maths/TP/TSM), and staple all the sheets together. (Failure to do that may result in misplaced/lost sheets, for which no responsibility can be taken by instructors.)

1. Let

$$\varphi(x_1, x_2, x_3) = \sin(x_1 - x_2)\sin(x_1 - x_3) + \sin^2 x_2 + c\sin x_3 \sin(2x_3) + \sin^3(x_1 + x_2 + x_3).$$

Furthermore, let A be the symmetric 3×3-matrix with entries $a_{ij} = \frac{\partial^2\varphi}{\partial x_i \partial x_j}(0,0,0)$.

(**a**) Write down the matrix A.

(**b**) Determine all values of the parameter c for which the corresponding quadratic form is positive definite.

(**c**) Does φ have a local minimum at the origin $(0,0,0)$ for $c = 1/5$?

2. (**a**) Assume that for a bilinear form b on $\mathbb{R}^n$ satisfies

$$b(e_i, e_j) = 1, \text{ for all } i, j,$$

where e_i are standard unit vectors. Compute the signature of this form.

(**b**) Same problem for the bilinear form such that

$$b(e_i, e_i) = 0, b(e_i, e_j) = 1 \text{ for } i \neq j,$$

where e_i are standard unit vectors.

3. Find an orthonormal basis of eigenvectors for the matrix

$$B = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

For the quadratic form $q(x) = (Bx, x)$, compute the maximal and the minimal value of $q(x)$ on the unit sphere $S = \{x \mid (x,x) = 1\}$.

4. Find a triangular change of basis (as in the proof of Jacobi theorem) where the matrix of the bilinear form associated to the matrix $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$ is diagonal.