MA 1112: Linear Algebra II
Tutorial problems, March 26, 2019

1. Compute the eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, and determine the signature of the quadratic form

$$q(x_1e_1 + x_2e_2 + x_3e_3) = x_1x_2 + x_2x_3.$$

2. Let

$$\varphi(x_1, x_2) = \sin^2(x_1 - x_2) - e^{\frac{1}{2}(x_1^2+x_2^2)+2cx_1x_2}.$$

Furthermore, let A be the symmetric 2×2-matrix with entries $a_{ij} = \frac{\partial^2\varphi}{\partial x_i \partial x_j}(0,0,0)$.

(**a**) Write down the matrix A.

(**b**) Determine all values of the parameter c for which the corresponding quadratic form is positive definite.

(**c**) Does φ have a local minimum at the origin $(0,0)$ for $c = -3/5$?

3. For the scalar product $(A, B) = \mathrm{tr}(AB^T)$ on the space of all $n \times n$-matrices, show that

$$(AB, AB) \leqslant (A, A)(B, B)$$

for all matrices A and B. (*Hint*: write (AB, AB) explicitly using the entries of A and B, and use the Cauchy–Schwartz inequality).

Optional question: Show that in $\mathbb{R}^n$, it is impossible to find $n+2$ vectors that only form obtuse angles (that is, $(v_i, v_j) < 0$ for all $i \neq j$).