MA2215: Fields, rings, and modules
Homework problems due on October 29, 2012

1. (**a**) Describe invertible elements in $\mathbb{Z}/12\mathbb{Z}$.

(**b**) Are $\overline{8}$ and $\overline{9}$ associates in $\mathbb{Z}/12\mathbb{Z}$?

(**c**) Show that in an integral domain R, if $a \mid b$ and $b \mid a$, then a and b are associates.

2. (**a**) Which elements of $\mathbb{Z}/12\mathbb{Z}$ are divisors of $\overline{6}$?

(**b**) Let R be an integral domain, and assume that two elements a and b in R have a greatest common divisor. Show that if d_1 and d_2 are two greatest common divisors of a and b, then d_1 and d_2 are associates.

3. Let R be a principal ideal domain. Show that the set $(a, b) := \{ax + by \colon x, y \in R\}$ is an ideal. Considering an element c that generates that ideal, show that two elements of R always have a greatest common divisor.

4. Plot on the plane the set of all multiples of the Gaussian integer $2 + i$, and compute the number of elements in the factor ring $\mathbb{Z}[i]/(2 + i)\mathbb{Z}[i]$.