MA2317: Introduction to Number Theory
Tutorial problems, October 22, 2010

1. Solve the system of congruences

$$x \equiv 1 \pmod{3},$$
$$x \equiv 5 \pmod{7},$$
$$x \equiv 9 \pmod{11}.$$

2. Which of the following systems of congruences do have integer solutions?

(a) $x \equiv 11 \pmod{84}$, $x \equiv 8 \pmod{36}$. **(b)** $x \equiv 11 \pmod{84}$, $x \equiv 5 \pmod{36}$. **(c)** $x \equiv 11 \pmod{84}$, $x \equiv 23 \pmod{36}$.

3. Compute the following Legendre symbols:
(a) $\left(\frac{23}{103}\right)$; **(b)** $\left(\frac{47}{101}\right)$; **(c)** $\left(\frac{253}{257}\right)$.

4. Which of the following congruences have solutions?
(a) $x^2 - 7x + 3 \equiv 0 \pmod{5}$;
(b) $x^2 + 2x - 9 \equiv 0 \pmod{97}$;
(c) $x^2 - 7x + 3 \equiv 0 \pmod{35}$?

5. In class, we proved that $\left(\frac{-1}{p}\right) = (-1)^{\frac{p-1}{2}}$. Using that, show that for every n all prime divisors of $4n^2 + 1$ are of the form $4k + 1$, and adapt the "$p_1 p_2 \cdots p_n - 1$"-argument proving the infinitude of primes to show that there are infinitely many primes of the form $4k + 1$.