MA3413: Group representations I
Homework problems due on October 18, 2012

In all the problems below, the ground field is the field of complex numbers.

1. Show that any irreducible representation of a finite abelian group is one-dimensional.

2. For each of the following cases, find a decomposition of the representation (V, ρ) of the group G into a direct sum of irreducible representations, and compute the dimension of the space of intertwining operators on it:

(**a**) $G = \mathbb{Z}/n\mathbb{Z} = \{e, g, g^2, \ldots, g^{n-1}\}$, $V = \mathbb{C}^n$, $\rho(g)$ is a cyclic shift of basis vectors: $\rho(g)e_1 = e_2$, $\ldots$, $\rho(g)e_n = e_1$;

(**b**) $G = S_n$, $V = \mathbb{C}^n$, $\sigma \in S_n$ permutes basis vectors accordingly: $\rho(\sigma)e_i = e_{\sigma(i)}$;

(**c**) $G = S_3$, the representation is its (left) regular representation.

3. Find in the dihedral group D_n (group of symmetries of the regular n-gon) two elements a, b that generate this group and satisfy relations $a^n = e$, $b^2 = e$, and $ba = a^{-1}b$.

4. Find all (equivalence classes of) 1-dimensional representations of (**a**) D_4; (**b**) D_5; (**c**) D_n; (**d**) Q_8 (quaternion units).

5. Find all (equivalence classes of) 2-dimensional representations of (**a**) $\mathbb{Z}/5\mathbb{Z}$; (**b**) D_4; (**c**) D_5; (**d**) Q_8.

6. Write down all irreducible characters for (**a**) $S_3 = D_3$; (**b**) D_4; (**c**) D_5; (**d**) Q_8. Check directly the orthonormality property for these characters.

Optional question (does not count towards the continuous assessment): Factor the Dedekind–Frobenius determinant (determinant of the multiplication table of the group that we discussed in the first lecture) for S_3, D_4, and Q_8 (you can use any computer software of your choice to do it). Can you guess how characters of irreps can be read from the factors of the determinant?