The main goal of this tutorial is to compute the character table of S_5:

#	1	10	20	15	30	20	24
cycle type	1^5	21^3	31^2	2^21	41	32	5
$V_5 \simeq \mathbb{1}$							
sgn							
$V_{4,1}$							
$V_{3,2}$							
$V_{4,1} \otimes \mathrm{sgn}$							
$V_{3,2} \otimes \mathrm{sgn}$							
$V_{3,1,1}$							

For that, we shall use the set representations $\mathbb{C}M_\lambda$ arising from the actions of S_n on the sets of Young tableaux.

1. Explain why $\mathbb{C}M_5$ is the trivial representation.

2. Compute the character of $\mathbb{C}M_{4,1}$, and explain how it decomposes into irreducibles (it contains the trivial representation and one more irreducible).

3. Compute the character of $\mathbb{C}M_{3,2}$, and explain how it decomposes into irreducibles (it contains the trivial representation, the representation from the previous question, and one more irreducible).

4. Tensoring the representations that you found with the sign representation, compute characters of three more irreducibles.

5. Compute the character of the last irreducible, either finding it as orthogonal to all other characters or decomposing $M_{3,1,1}$, or decomposing $M_{1,1,1,1,1}$ (which is the regular representation, so the missing representation occurs there with the multiplicity equal to dimension.