MA3413: Group Representations I
Tutorial questions, February 11, 2015

The main goal of this tutorial is to describe characters of all complex irreducible representations of S_4. We already know that there are two one-dimensional representations, the trivial representation and the sign representation.

1. Describe conjugacy classes of S_4.

2. Recall that the representation of S_4 in $\mathbb{C}^4$ by permutations of basis vectors is isomorphic to a direct sum of the trivial representation and a three-dimensional irreducible representation, that we shall denote by (V, ρ). Compute the character of V.

3. Show that the representation $(V, \rho \otimes \text{sign})$ where each element g acts by $\text{sign}(g)\rho(g)$ is irreducible and not isomorphic to (V, ρ).

4. Find a surjective homomorphism from S_4 to S_3. Explain how to use it to construct a two-dimensional irreducible representation U.

5. Write down the character table for S_4.

6. (If there is time left) Show that V is an irreducible representation of the group A_4 of even permutations of four elements, and describe all other complex irreducible representations of A_4 and their characters.