MA 3419: Galois theory
Homework problems due November 14, 2017

Solutions to this are due by the end of the 11am class on Tuesday November 14. Please attach a cover sheet with a declaration `http://tcd-ie.libguides.com/plagiarism/declaration` confirming that you know and understand College rules on plagiarism.

Please put your name and student number on each of the sheets you are handing in. (Or, ideally, staple them together).

1. Explain how to compute $\cos(2\pi/13)$ by solving quadratic and cubic equations only.

2. Find the splitting field and the Galois group of $x^3 - 5$ over $\mathbb{Q}(\sqrt{2})$.

3. Find the splitting field and the Galois group of $x^4 - 2x^2 - 5$ over $\mathbb{Q}$.

4. Let K be the splitting field of $f = x^4 - 2$ over $\mathbb{Q}$. In the previous homework you established that $G = \mathrm{Gal}(K\colon \mathbb{Q})$ is isomorphic to D_4. List all the subgroups of G and use this to write down all the intermediate fields between $\mathbb{Q}$ and K. Explain which of those intermediate fields are Galois extensions of $\mathbb{Q}$.

5. Compute the Galois groups of the splitting fields of the polynomial $x^4 - 3$ over $\mathbb{F}_5$, $\mathbb{F}_7$, $\mathbb{F}_{11}$ and $\mathbb{F}_{13}$.

6. Let $k \subset K$ be a Galois extension with the Galois group $G = \{g_1, \ldots, g_n\}$, and let $a \in K$. Show that $K = k(a)$ if and only if $g_1(a), \ldots, g_n(a)$ are distinct elements of K.