

Assignment 1

MA341C — Seminar on *Proofs from THE BOOK*

Trinity College Dublin

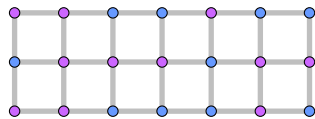
NAME AND SURNAME:

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Note: solutions to this assignment are due by 11am on Wednesday, September 19th. Please attach a cover sheet with a declaration (<http://tcd-ie.libguides.com/plagiarism/declaration>) confirming that you know and understand College rules on plagiarism. All exercises are weighed equally unless otherwise stated.

Exercise 1. Show, by providing an example, that for any choice of natural numbers $m, n \geq 2$ there is a sequence of mn real numbers with no decreasing subsequence of length $m + 1$ nor any increasing subsequence of length $n + 1$.

Exercise 2. Prove that if we draw a 3×7 grid as in the figure below, with vertices coloured purple and blue, there is one monochromatic rectangle with its edges parallel to the grid.



Exercise 3. Fix a natural number n and let σ be a permutation on n letters. We say $i \in [n]$ is a fixed point of σ if $\sigma(i) = i$, and write $\text{fix}(\sigma)$ for the number of such fixed points. Use double counting to show that

$$\frac{1}{n!} \sum_{\sigma \in \mathcal{S}_n} \text{fix}(\sigma) = 1.$$

That is, the expected number of fixed points of a permutation is 1.

Exercise 4. Suppose that t is a real number and N is a positive integer. Show that among the N numbers $t, 2t, \dots, Nt$ there is at least one which differs by at most $1/N$ from an integer.