Assignment 9

MA341C — Seminar on *Proofs from THE BOOK*

Trinity College Dublin

NAME AND SURNAME: .....

STUDENT NUMBER: ..... NUMBER OF PAGES: .....

Note: solutions to this assignment are due by 11am on Wednesday, November 21st. Please attach a cover sheet with a declaration (http://tcd-ie.libguides.com/plagiarism/declaration) confirming that you know and understand College rules on plagiarism. All exercises are weighed equally unless otherwise stated.

Recall that a finite graph G is k-regular if each vertex has degree k, and G is regular if it is k-regular for some positive number k.

Exercise 1. Let $k \geq 2$, and define polynomials $p_0(x) = 1$, $p_1(x) = x$, $p_2(x) = x^2 - k$, and

$$p_l(x) = x p_{l-1}(x) - (k-1) p_{l-2}(x)$$

for all $l \geq 3$. Show that if A is the adjacency matrix of a k-regular graph G then the entry $(p_l(A))_{ij}$ is the number of walks of length l in G that start at v_i, end at v_j, and have any two consecutive edges distinct.

Exercise 2. Let A be the adjacency matrix of a finite graph G on n vertices. Prove that the $n \times n$-matrix J, whose entries are all one, is a polynomial in A if and only if G is regular and connected.

Exercise 3. The algorithm described in Lemma 2 of Chapter 36 (The Dinitz Problem) always results in a stable matching. However, there can be many stable matchings. Show that the algorithm favours the side who proposes: a man always ends up with the highest ranked partner amongst possible stable matchings.

Exercise 4. Suppose that for a set of N points in the two-dimensional plane, the pairwise distances of all points are greater than 1. Prove that it is possible to choose $N/7$ of those points for which all pairwise distances are greater than $\sqrt{3}$.