MA341D Homework assignment 1

Due in class on February 5, 2018

1. (a) Let $R = \mathbb{C}[x, y, z]$ equipped with the `lex` order with $x > y > z$. Compute the S-polynomial of the polynomials $2x+3y+z+x^2-z^2+z^3$ and $2x^2y^8-3x^5yz^4+xyz^3-xy^4$.

   (b) Same task if we consider the `glex` order with $x > y > z$.

2. For each of the following systems of polynomials f_1, f_2, f_3, compute a Gröbner basis for the ideal (f_1, f_2, f_3). Use the `lex` order with $x > y > z$.

   (a) $f_1 = x^3yz - xz^2$, $f_2 = xy^2z - xyz$, $f_3 = x^2y^2 - z$;

   (b) $f_1 = x^2y + xz + y^2z$, $f_2 = xz^2 - yz$, $f_3 = xyz - y^2$;

   (c) $f_1 = xy^2 - z - z^2$, $f_2 = x^2y - y$, $f_3 = y^2 - z^2$.

3. (a) Show that the polynomial $xy^3-z^2+y^5-z^3$ belongs to the ideal $(-x^3+y, x^2y-z) \subset \mathbb{C}[x, y, z]$.

   (b) Show that the polynomial x^3z-2y^2 does not belong to the ideal $I = (xz-y, xy+2z^2, y-z) \subset \mathbb{C}[x, y, z] = R$, and represent the coset of that polynomial in R/I as a linear combination of normal monomials.

4. You are required to pay 1 dollar and 67 cents in coins, using pennies (1 cent), nickels (5 cents), dimes (10 cents) and quarters (25 cents). You have to pay the exact amount, no change is available. Use Gröbner bases to determine the way to make the payment using the smallest possible number of coins. (*Hint*: in the polynomial ring $\mathbb{C}[p, n, d, q]$, consider the ideal generated by the polynomials $p^5 - n$, $p^{10} - d$, $p^{25} - q$. For the `glex` order, find the normal form of p^{167} relative to that ideal, i.e. write p^{167} using only normal monomials with respect to the Gröbner basis, and explain why it is useful for this question.)