Benchmarking of global full-f gyrokinetic codes

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Outline

• Introduction
  • Difficulty for benchmarking full-f gyrokinetic codes
  • GYSELA and GT5D codes

• Benchmarking for gyrokinetics
  • Linear Ion temperature gradient (ITG) modes
  • Nonlinear decaying ITG turbulence

• Benchmarking for neoclassical physics
  • Neoclassical poloidal velocity and heat diffusivity
  • Zonal flow damping

• Preliminary results for flux-driven case

• Summary and future plan
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Benchmarking of gyrokinetic codes

Benchmark for Delta-f simulation [1]

- Time evolution of Delta-f distribution function
- Benchmark for ITG turbulence is almost done

Benchmark for Full-f simulation

- Time evolution of Delta-f + Equilibrium distribution function
- Gyrokinetics + Neoclassical based on first principle [2]
- Few benchmarking between Full-f codes [3], especially for flux driven case

### Delta-f vs Full-f

<table>
<thead>
<tr>
<th>Physics</th>
<th>Delta-f</th>
<th>Full-f</th>
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</thead>
<tbody>
<tr>
<td>Linear instability (ITG modes)</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>ITG turbulence</td>
<td>✔️</td>
<td>✔️</td>
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<tr>
<td>Electron-scale turbulence</td>
<td>✔️</td>
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<td>Electromagnetic turbulence</td>
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<td>Zonal flow damping</td>
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<td>Collisional transport</td>
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<td>Avalanche like transport property</td>
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<tr>
<td>Interaction between turbulence and neoclassical</td>
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</table>

- Full-f gyrokinetic simulations include too many physics working together
  - Benchmarking subproblems at ideal limit
Decomposing into subproblems

Gyorkinetics (collisionless limit)
- Linear benchmarking (growth rate and real frequency)
- Nonlinear ITG decaying turbulence (Nonlinear critical gradient level)

Neoclassical (absence of instabilities)
- Neoclassical prediction for heat diffusivity
- Neoclassical prediction for poloidal velocity
- Zonal flow damping and residual level

Flux driven ITG turbulence
- Avalanche like turbulence (interaction with background)
- Interactions of neoclassical and turbulent transports
Governing Equations

Vlasov eq.

\[ \frac{\partial \mathcal{J} f}{\partial t} + \nabla \cdot \left( \mathcal{J} \dot{R} f \right) + \frac{\partial}{\partial v_\parallel} (\mathcal{J} \dot{v}_\parallel f) = \text{RHS} (f) \]

**GT5D:** \[ \text{RHS} (f) = \mathcal{J} [C (f) + S_{\text{src}} + K(f)] \]

**GYSELA:** \[ \text{RHS} (f) = \mathcal{J} [C (f) + S_{\text{src}} + K(f) + D_r (f)] \]

LHS is the same for GT5D and GYSELA, GYSELA has buffer region

Poisson eq.

\[ -\nabla_\perp \cdot \frac{\rho_{ti}^2}{\lambda_{Di}^2} \nabla_\perp \phi + \frac{1}{\lambda_{De}^2} (\phi - \langle \phi \rangle_f) = 4\pi \left[ e_i \int d^6 Z \delta f \delta (\mathbf{R} + \rho - \mathbf{x}) \right] \]

Poisson equation with long-wavelength approximation

**GT5D:** 40 points for gyroaveraging

**GYSELA:** 8 points for gyroaveraging
**Code Summary**

### Numerical schemes

**GT5D [1]:** Vlasov (Finite Difference) + Poisson (Finite Element)

**GYSELA [2]:** Vlasov (Semi-Lagrangian) + Poisson (Finite Difference)

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**Boundary condition (potential)**

**GT5D [1]:** \( \mathbf{x} = (\psi, \theta, \varphi) \)

**GYSELA [2]:** \( \mathbf{x} = (r, \theta, \varphi) \)

- Dirichlet: \( \phi = 0 \) at the magnetic axis
- Neumann: \( \partial_r \phi = 0 \) at the magnetic axis

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- **Different boundary condition** imposed at the **magnetic axis** could affect large mode structure in radial direction
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Equilibrium profiles (Turb)

- Based on Cyclon Base Case (CBC) parameters (at mid-minor radius)

<table>
<thead>
<tr>
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- Electron temperature profile is the same as ion temperature profile
- \( \rho^{-1}_* = 150 \) (Linear and Decaying turbulence)
**Linear Benchmark**

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Initial equilibrium: Local Maxwellian

$$f_{LM} = F_M(\psi, \epsilon)$$

$n = 15$ mode

- Good agreements in growth rate and real frequency
Decaying Turbulence

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<tr>
<th></th>
<th>$N_R$ ($N_r$)</th>
<th>$N_Z$ ($N_\theta$)</th>
<th>$N_\varphi$</th>
<th>$N_{v\parallel}$</th>
<th>$N_\mu$</th>
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<tr>
<td><strong>GT5D</strong></td>
<td>200</td>
<td>200</td>
<td>128</td>
<td>128</td>
<td>16</td>
</tr>
<tr>
<td><strong>GYSELA</strong></td>
<td>256</td>
<td>1024</td>
<td>128</td>
<td>128</td>
<td>16</td>
</tr>
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Initial equilibrium: Local Maxwellian

$$f_{LM} = F_M(\psi, \epsilon)$$

Initial condition:

$$\overline{F}_s = \overline{F}_{s,eq} \left( 1 + \epsilon \sum_{n=1}^{n_{\text{max}}} \sum_{m=m_{\text{min}(n)}}^{m_{\text{max}(n)}} \cos(m\theta + n\varphi + \delta_{mn}) \right)$$

$$m_{\text{min}} = nq - \delta_m \quad m_{\text{max}} = nq + \delta_m$$

- Simulations without collision terms to avoid neoclassical effect
- Adding perturbations to unstable toroidal modes with no-filtering
Time evolution of potential

\[ \rho = 0.5 \]

- Except for the nonlinear saturation level, reasonable agreements are obtained
Potential Contours

GYSELA
\[ \Phi_m - \Phi_0 \quad tR_0/v_{ti} = 60 \]

Fine structures are found in GT5D compared to GYSELA.

Possibility due to the few number of points for gyro average in GYSELA.
Time evolution of flux (decay)

- Radial average for $\rho = 0.45 \sim 0.55$
- Agreements in temperature gradient and transport level in the steady state
Spatio-temporal structure

GYSELA $v E \times B / v_{ti}$

GT5D

$R_0 / L_{ti}$

$\chi_i / (\rho_{ti}^2 v_{ti} / R_0)$

- Same time scale for the temperature relaxation
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Neoclassical poloidal velocity

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Initial equilibrium: Local Maxwellian

\[ f_{LM} = F_M(\psi, \epsilon) \]

- Large density gradient to **suppress instabilities**
- Good agreement with H-H’s analytical formula [1]
- Coupled with radial Force balance

Code: \[ E_r^{GYSA} = -\nabla \phi \]

H-H: \[ E_r^{H-H} - v_\phi B_\theta + v_\theta B_\phi = \frac{\nabla P}{ne} \]

\[ v_\theta = k \frac{\nabla T}{eB} \quad \text{(H-H formulae)} \]

Good agreements in numerical and analytical results

Compared with Chang-Hinton’s theory [1] (radial averaged for rho = 0.4-0.6)

\[ q_i = -K_2 n_i \epsilon^{1/2} \left( \rho_{t \theta}^2 / \tau_i \right) \frac{\partial T_i}{\partial r} \]

\[ K_2 = K_2^{(0)} \left( \frac{K_2^* / 0.66}{1 + a_2 \nu_{*i}^{1/2}} + \frac{(c_2/b_2) \nu_{*i}^{3/2}}{1 + c_2 \nu_{*i}^{3/2}} F \right) \]

Good agreements in numerical and analytical results

Rosenbluth-Hinton test

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Initial equilibrium: $\overline{F}_s = \overline{F}_{s,eq} \left(1 + \epsilon \sin \left(2\pi \frac{(r - r_{min})}{2L_r}\right)\right)$

Analytical result by Sugama and Watanabe [1]

$$\phi_{00}(t) = \phi_{00}(\infty) + [\phi_{00}(0) - \phi_{00}(\infty)] \cos(\omega_G) \exp(\gamma t)$$

Residual level is computed by RH result [2]

$$\phi_{00}(\infty) = \phi_{00}(0) / \left(1 + 1.6q^2/\epsilon^{1/2}\right)$$


In relation to A. Boltino’s talk (Monday)

- Flat radial profile to mimic **local limit** (theories are established at local limit)
- Good agreements with theoretical results
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Spatio-temporal structure

Smaller scale zonal flow structures are found in GT5D
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Conclusion

Benchmarking for flux-driven Full-f codes

Gyrokinetics  Collisionless limit

- Good agreements in linear properties of ITG mode
- Good agreements in time evolution of flux and temperature gradient in decaying turbulence
- Some differences in fine scale mode structure

Neoclassical  Without turbulence (axisymmetric limit)

- Good agreements with theoretical prediction of neoclassical heat flux and neoclassical poloidal velocity
- Good agreements with theoretical description of zonal flow damping and its residual level

Fluxdriven

- Preliminary results showing different structures in zonal flow

Future work

- Fluxdriven benchmarking with exactly same source and sink terms