Comparison of ballooning modes in a global gyrokinetic code, ORB5, with an MHD code, MISHKA

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This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 under grant agreement No 633053.
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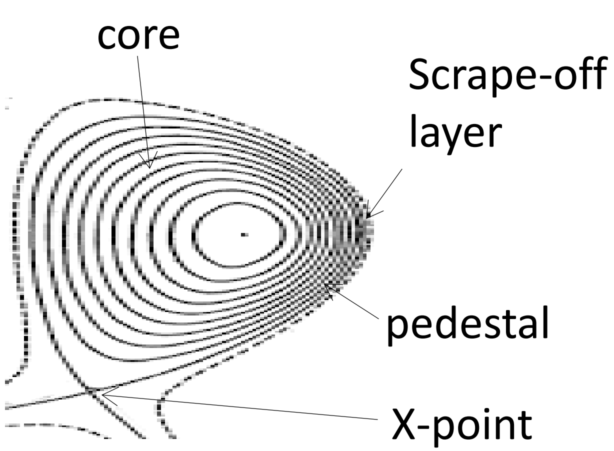
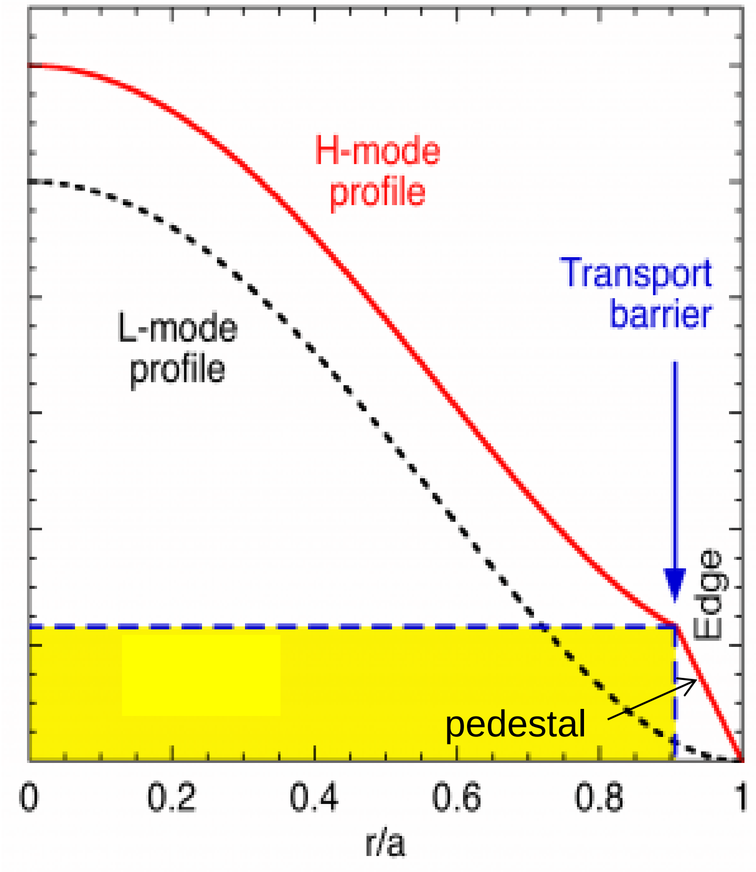
Pedestal

- region with suppression of turbulence between SOL (scrape-off layer) and core





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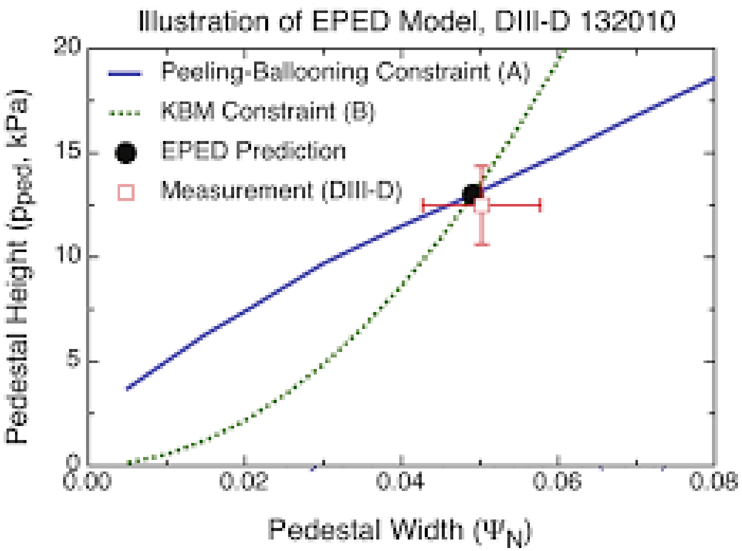
EPED Model

- Predictive Model for pedestal height and width
- Both increase until ELM is triggered
- Onset of KBM turbulence provides second relationship between height and width
- Leads to relationship between the pedestal width and the plasma poloidal beta at the top of the pedestal



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Gyrokinetic Model(1)

- distribution function, *f*, describes the number of particles per unit volume in phase space:

$$N = \int f \, d\bar{v} d\bar{R}$$

where *N* is the number of particles in any phase space volume of the plasma and the right hand side is integrated over that volume

- time development is then given by the Vlasov equation:

$$\frac{\partial}{\partial t} f + \frac{d\bar{R}}{dt} \frac{\partial}{\partial \bar{R}} f + \frac{d\bar{v}}{dt} \frac{\partial}{\partial \bar{v}} f = 0$$

where any collisional terms, sources and sinks are added to the right hand side of the equation

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Gyrokinetic Model(2)

- the distribution function, f, is rewritten in terms of $\theta, \mu, \bar{R}, p_{||}$ and t, where $\mu = \frac{mv_\perp^2}{2B}$ is the magnetic moment and $p_{||} = mv_{||} + qA_{||}$ is the canonical parallel momentum
- f is taken to be independent of θ, leaving the gyrokinetic Vlasov equation:

$$\frac{\partial}{\partial t}f + \frac{dp_{||}}{dt}\frac{\partial}{\partial p_{||}}f + \frac{d\bar{R}}{dt}\frac{\partial}{\partial \bar{R}}f = 0$$

where $\frac{d\bar{R}}{dt}$ is due to drifts

- this equation is solved alongside the equations of motion and the gyrokinetic Maxwell's equations

$$-\nabla.\frac{n_0 mc^2}{B^2}\nabla_\perp \phi = \sum_{sp}\int dWef$$ Poisson's Equation

$$\sum_{sp}\int dW\left(\frac{ep_{||}}{mc}f - \frac{e^2}{mc^2}A_{||}f_0\right) + \frac{1}{4\pi}\nabla_\perp^2 A_{||} = 0$$ Ampere's Law

- control variates scheme is used where f is split between a time independent part f_0 and a time dependent part δf

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ORB5

- solves multiple species Vlasov equations interacting with electromagnetic perturbations in a global tokamak geometry, allowing for collisions and various source terms

- the Particle-In-Cell method, PIC method, is used in ORB5
- δf is discretized as

$$\delta f = \frac{N_{ph}}{N} \sum_{p=1}^{N} \frac{1}{2\pi B_{||}^{*}} w_p(t) \delta\left(\bar{R} - \bar{R}_p(t)\right) \mathrm{x}\, \delta\left(\mathrm{v}_{||} - v_{||p}(t)\right) \delta(\mu - \mu_p(t_0))$$

- each marker is characterized by its weight $w_p(t)$ and its location $\bar{R}_p(t), v_{||p}(t)$ and $\mu_p(t_0)$ in phase space
- temporal evolution of δf is given by:

$$\frac{dw_p(t)}{dt} = \frac{N}{N_{ph}} \tau(\bar{E})_p \Omega_p$$

A global collisionless PIC code in magnetic coordinates S.Jolliet, A.Bottino et al.(2007)

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Cancellation Problem

- arises from the choice of $p_{||}$ formalism for gyrokinetics
- $p_{||}$ formalism is chosen for ORB5 since it is easier to obtain equations of motion
  - in $v_{||}$ formalism, the current is calculated as first moment of the distribution
  - in $p_{||}$ formalism, to calculate the currents you have to get $A_{||}$ since $p_{||} = mv_{||} + qA_{||}$
- for calculating $A_{||}$ from Ampere's Law, there is one term that must be calculated numerical from the discretization of particles and another term that is calculated from the grid which should cancel with each other approximately
- this leads to an inaccurate evaluation of $A_{||}$
- there is a method to avoid this problem which involves using the adiabatic part of the perturbed distribution function and modifying the weights of the markers
- for more info check: http://www.fusione.enea.it/EVENTS/eventifiles/EFTC14-2011/Orals/26sett/Hatzky.pdf

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GS2

- local gyrokinetic code
- for Vlasov equation, the velocity-space coordinates are (E, μ) where $E = \frac{1}{2}mv^2$ is the kinetic energy and μ is as defined previously
- to get $v_{||}$ the following relationship is used:

$$v_{||}^2 = \frac{2(E - \mu B)}{m}$$

- works on a single flux surface
- assumes an infinite space with constant pressure gradient, temperature gradient, e.t.c.

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MHD Theory(1)

- Only applicable when:

The plasma is strongly collisional, so that the time scale of collisions is shorter than the other characteristic times in the system (Maxwellian distributions)

Ideal MHD - Resistivity due to collisions is small, so magnetic diffusion times over any scale length present in the system must be longer than any time scale of interest

System is smooth and slowly evolving, so length scales are much longer than the ion skin depth and Larmor radius perpendicular to the field (Landau damping) and time scales much longer than the gyration time

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MHD Theory(2)

- equations of MHD are:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho\bar{v}) = 0$$ Continuity equation

$$\rho\left(\frac{\partial}{\partial t} + \bar{v}.\nabla\right)\bar{v} = \bar{J}\mathrm{x}\bar{B} - \nabla p$$ Momentum equation

$$\bar{E} + \bar{v}\mathrm{x}\bar{B} = 0$$ Ideal Ohm's Law

$$\frac{\partial \bar{B}}{\partial t} = -\nabla \mathrm{x} \bar{E}$$ Faraday's Law

$$\mu_0\bar{J} = \nabla\mathrm{x}\bar{B}$$ Low-frequency Ampere's Law

$$\nabla.\overline{\mathrm{B}} = 0$$ Magnetic Divergence Constraint

$$\frac{d}{dt}\left(\frac{p}{\rho^{\gamma}}\right) = 0$$ Energy equation

$$\nabla.v = 0$$ Incompressibility (Incompressible MHD)

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MHD instabilities

- MISHKA solves the stability problem
- An instability occurs when the potential energy is negative, since energy is conserved
- Derived from the linearized MHD equations

$$\delta W = \delta W_{plasma} + \delta W_{vacuum}$$

$$\delta W_{vacuum} = \frac{1}{2}\int_{vac} d\mathbf{r}\, \frac{|\mathbf{B}_1|^2}{\mu_0}$$ (vacuum field line bending, always stabilising)

Stabilising: **Field line bending** | **Compression of magnetic field lines** | **Plasma compression**

$$\delta W_{plasma} = \frac{1}{2}\int_{plasma} \left[\frac{|\mathbf{B}_{1,\perp}|^2}{\mu_0} + \frac{B_0^2}{\mu_0}|\nabla\cdot\xi_\perp + 2\xi_\perp\cdot\kappa|^2 + \gamma p_0|\nabla\cdot\xi|^2 \right.$$

$$\left. - 2(\xi_\perp\cdot\nabla p_0)(\kappa\cdot\xi_\perp^*) - j_\parallel(\xi_\perp^*\times\mathbf{b})\cdot\mathbf{B}_{1,\perp} \right]$$

Destabilising: **Pressure driven modes** | **Current driven modes**

Stability code: Find ξ that minimises δW.

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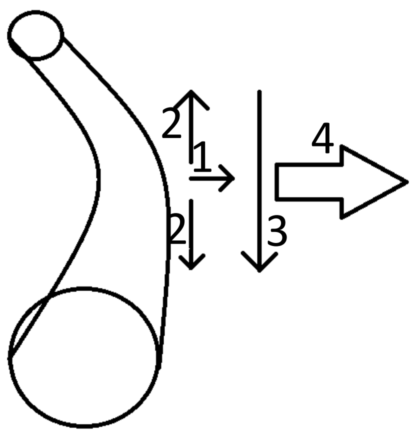
Ballooning Mode

Kinetic

MHD

Why does the field line bend?

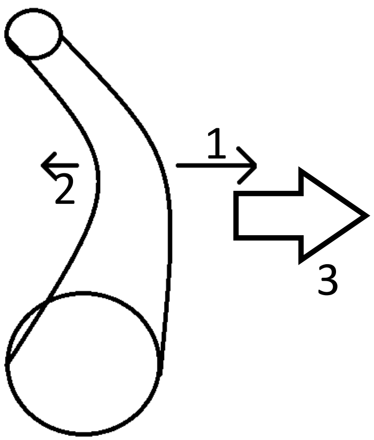


Electrostatic field creates current along field line, which creates a perturbed magnetic field (results in $E_{||} \approx 0$)

1. ∇B force in the outward direction
2. Drifts for electrons and ions in opposite directions
3. *E* field develops
4. Causes *ExB* drift in the outward direction

From the frozen flux condition:

$$\frac{d\bar{B}}{dt} = -\nabla \mathrm{x} \bar{E}$$



1. Outward forces include particle pressure and ∇B force
2. Inward force is the force due to field tension
3. If outward forces are greater than inward forces this cause an over all outward force

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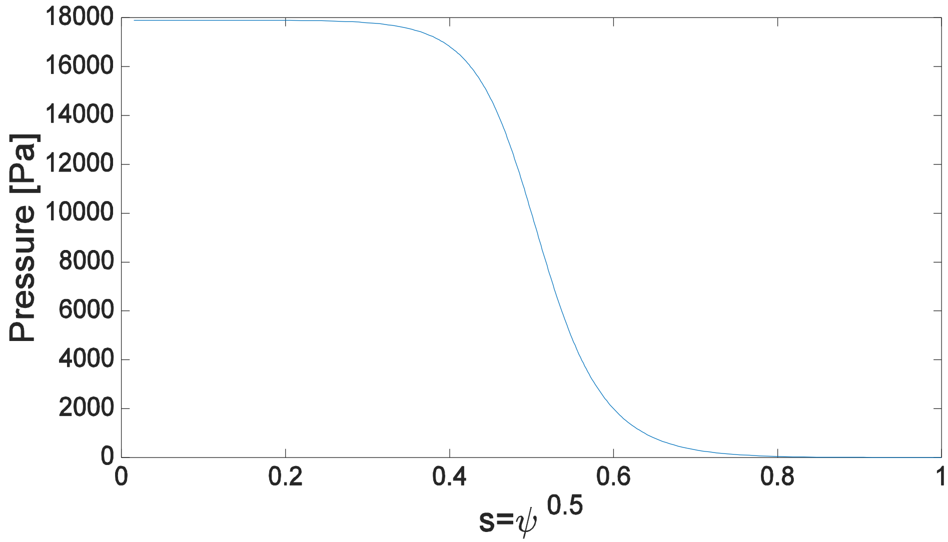
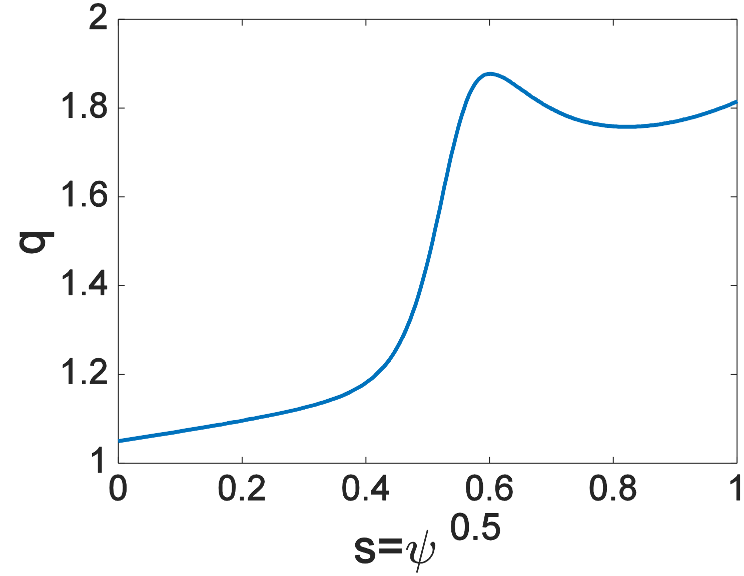
Equilibrium(1)





- equilibrium is motivated by the pedestal case
- pedestal moved inward to negate boundary effects

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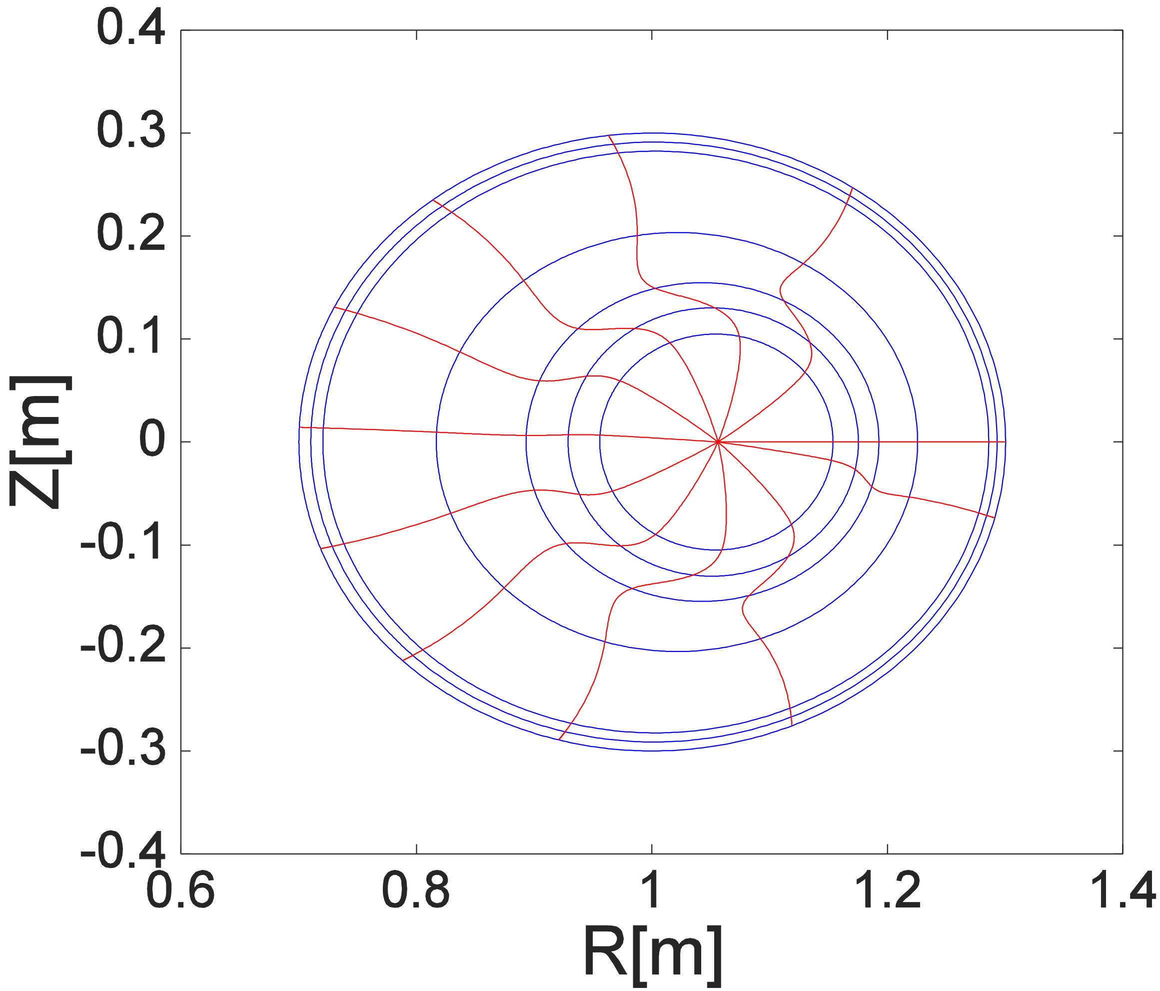
Equilibrium(2)



- simplified numerical equilibrium with circular outer boundary
- substantial shaping
- local shear at $s \approx 0.5$ and $\theta^* = 0$ small due to combination of global shear and local Shrafranov shift

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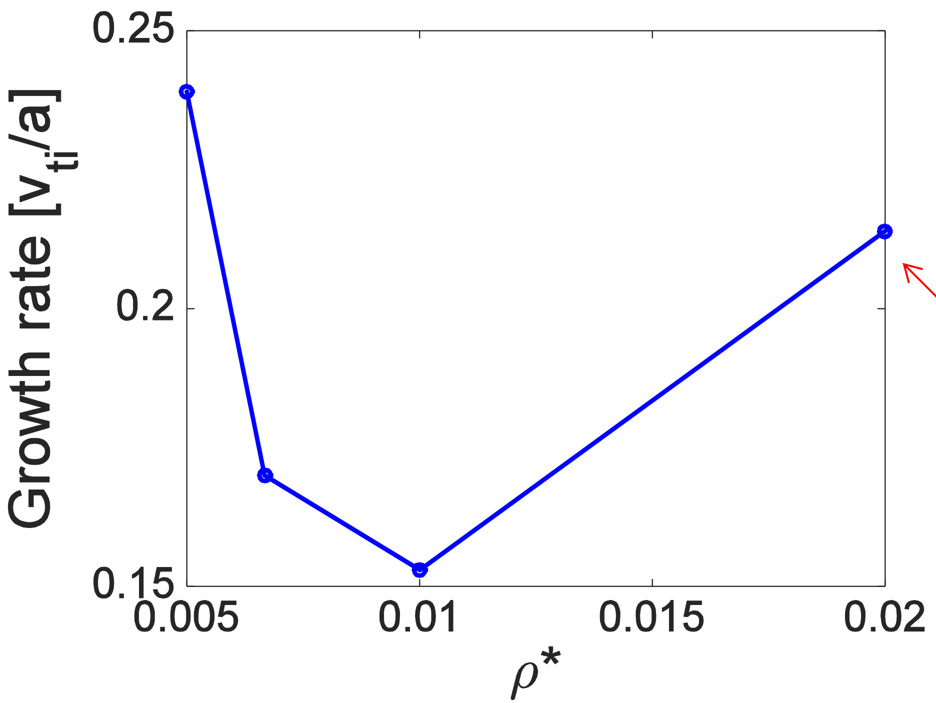
ρ^* scan



ORB5

- constant *n=20*
- increasing growth rate as ρ^* decreases
- expect growth rate to increase as $\rho^* \to 0$ and approach the MHD result
- increasing growth rate for largest ρ^*(could be due to a different mode, possibly ITG, but further analysis is necessary)

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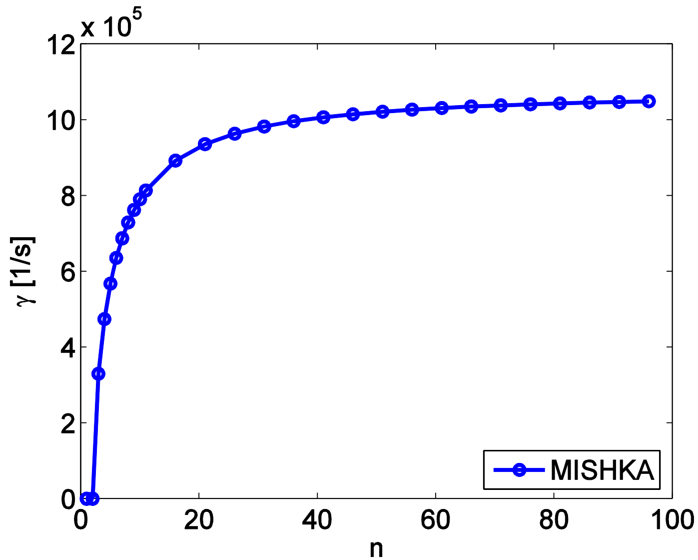
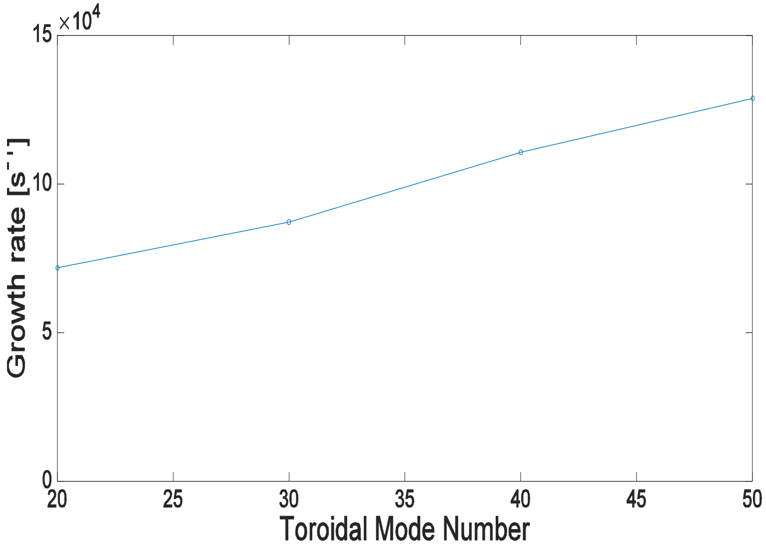
N scan



MISHKA



ORB5

- disagreement currently
- could be due to normalisation factors or mass ratios (further analysis is needed)

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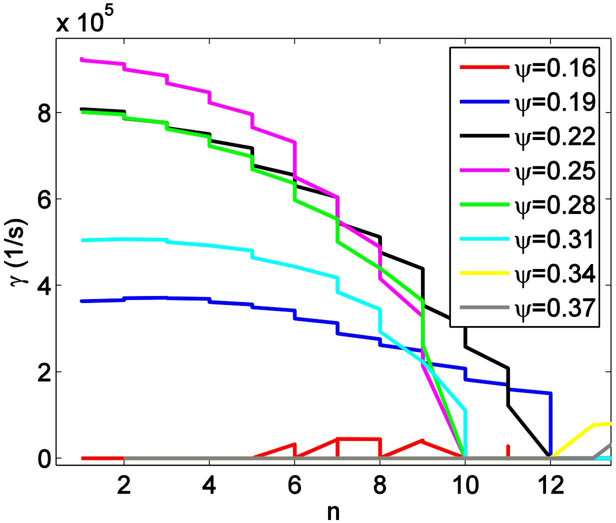
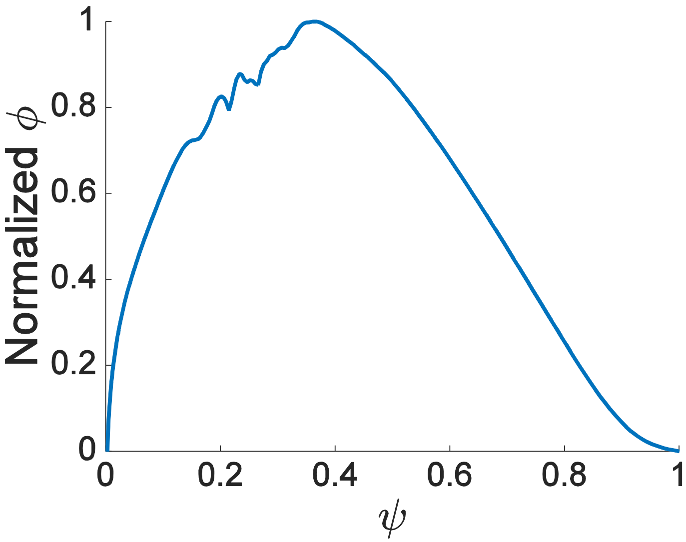
N scan(2)



GS2



ORB5

- global simulations models the stabilisation due to finite width effects so KBMs are not present
- these modes can be seen in the local gyrokinetic simulations since these finite width effects are not present

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Conclusion

- simplified equilibria motivated on the pedestal case, but moved in to avoid boundary effects
- increasing growthrate as ρ^* decreases as expected
- growthrate for ORB5 is about an order of magnitude smaller than for MISHKA
- small n KBMs are stabilized as expected

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