Entropy in Mathematics and in Physics
The 92nd Encounter between Mathematicians and Theoretical Physicists

Program

September 26–28, 2013

Strasbourg

1 Schedule

Thursday

9h30 *Nessim Sibony (Orsay)* – Entropy in holomorphic dynamics
10h30 Coffee break
11h00 *Rémi Peyre (Nancy)* – Free energy functional in an optimal-transportation setting
12h00 Lunch
15h30 *Anders Karlsson (Genève)* – A heat kernel approach to tree entropy
16h30 Coffee break
17h00 *Gérard Besson (Grenoble)* – On open 3-manifolds
18h00 Departure for the dinner at restaurant "Le petit bois vert", (2 quai de la Bruche, Petite France). All participants are invited.

Friday

9h30 *Karl-Theodor Sturm (Bonn)* – Ricci curvature and Entropy
10h30 Coffee break
11h00 *Freddy Bouchet (Lyon)* – Large deviations and averaging for stochastic partial differential equations describing atmosphere dynamics
12h00 Lunch
14h00 *Stéphane De Gérando (Paris)* – Introduction au Labyrinthe du temps
14h30 *Constantin Vernicos (Montpellier)* – Volume entropy of Hilbert geometries and approximability of convex bodies
15h30 Coffee break
16h00 *Gabriel Rivière (Lille)* – Eigenmodes of the wave equation, chaotic dynamical systems and entropy
17h00 *Djalil Chafaï (Paris)* – Some probabilistic aspects of Bolzmann entropy
18h00 Departure for the reception at the city hall. All participants invited.

Saturday

10h00 *Mauro Carfora (Pavia)* – The Ubiquitous Entropy: from Geometrical Enumeration to Quantum Field Theory
11h00 Coffee break
11h30 *Christian Maes (Leuven)* – Nonequilibrum entropies

2 Abstracts

Gérard Besson (Université Joseph Fourier)
On open 3-maniflods

Freddy Bouchet (École Normale Supérieure de Lyon)
Large deviations and averaging for stochastic partial differential equations describing atmosphere dynamics.

We consider the formation of large scale structures (zonal jets and vortices), in planetary atmosphere turbulence, within the barotropic quasi-geostrophic equations (the Ising model of geophysical turbulence). This model includes as a special case the 2D stochastic Navier-Stokes equations. We develop a theory based on tools of statistical physics (large deviations, averaging, kinetic theory) and field theory (instantons). We study the limit of a time scale separation between inertial dynamics on one hand, and the effect of forces and dissipation on the other hand. Using a kinetic theory approach, we prove that stochastic averaging can be performed explicitly in this problem, which is unusual in turbulent systems. It is then possible to integrate out all fast turbulent degrees of freedom, and to get explicitly an equation that describes the slow evolution of zonal jets. For some range of parameters, the dynamics has several attractors, with extremely rare and abrupt transition from one attractor to another. Using large deviation results, derived either from path integrals or from generalization of Freidlin–Wentzell theory, we compute the transition rates and transition trajectories between two attractors.

Mauro Carfora (Università di Pavia)
The Ubiquitous Entropy : from Geometrical Enumeration to Quantum Field Theory

This talk is designed for a general audience. I will review the ubiquitous and somewhat unexpected role that entropy plays in a number of geometrical and physical situations.

Djalil Chafaï (Université Paris-Dauphine)
Some probabilistic aspects of Boltzmann entropy.

Starting from the motivation of Boltzmann, we will follow the entropy along the central limit theorem of classical and of free probability, and we will end up with random matrices and Coulomb type gases.

Stéphane De Gérando (Paris)
Introduction au labyrinthe du temps.

Le labyrinthe du temps est à la fois une œuvre dynamique ("work in continuous progress") poly-artistique et technologique (image, son) et un projet de recherche en cours de développement. Stéphane de Gérando présentera

l'œuvre ainsi que des pistes pour des collaborations possibles avec des mathématiciens, sur le thème de cette pensée labyrinthique, fragmentation de la mémoire jusqu'au seuil entropique dune œuvre chaotique ou "décomposée".

Stéphane de Gérando, auteur du Labyrinthe du temps est compositeur. Il enseigne la composition et les nouvelles technologies au Conservatoire du XIXe arrondissement de Paris, docteur HDR, Premier Prix et 3eme Cycle de composition du CNSMDP, ancien directeur du CFMI de Strasbourg (département Arts de l'UdS) et du département musique du CEFEDEM d'Aquitaine.

Anders Karlsson (Université de Genève)
A heat kernel approach to tree entropy

The number of spanning trees of graphs is an invariant of interest in electrical networks, statistical physics, theoretical chemistry and mathematics. I will discuss the asymptotics of the number of spanning trees in families of graphs, especially discrete tori. The constant in the lead term is called the tree entropy. This number as well as other constants in the asymptotics have interest in number theory: they are expressed in terms of certain Mahler measures, zeta functions, modular forms, and their special values. When there is a limiting continuous torus, its determinant of Laplacian appears. Based on joint work with G. Chinta and J. Jorgenson. I will also mention recent works by F. Friedli and J. Louis.

Christian Maes (Katholieke Universiteit Leuven)
Nonequilibrium entropies

In contrast to the quite unique entropy concept useful for systems in (local) thermodynamic equilibrium, there is a variety of quite distinct nonequilibrium entropies, reflecting different physical points. We disentangle these entropies as they relate to heat, fluctuations, response, time-asymmetry, variational principles, monotonicity, volume contraction or statistical forces. However, not all of those extensions yield state quantities as understood thermodynamically. At the end we sketch how aspects of dynamical activity can take over for obtaining an extended Clausius relation.

Rémi Peyre (Université de Lorraine)
Free energy functional in an optimal-transportation setting

We consider a toy model for gravitational spontaneous symmetry breaking: in this model, the distribution of matter evolves according to a first-order equation, with a short-range attractive force which tends to make matter collapse, versus a diffusion term which tends to homogeneize density. It can be shown that this evolution is tantamount to the gradient flow of the free energy (hence involving entropy) in some infinite-dimensional Riemannian manifold defined in terms of optimal transportationall of that shall be explained in detail. Hence our goal is to understand the behaviour of the free energy functional, in particular near the

critical point (that is, when diffusion is just sufficient to prevent spontaneous symmetry breaking). We will show that the critical exponant of the activation energy is $(3 - n/2)_+$, n being the dimension of the space. This involves finding sharp estimates, both below and above, on the entropy of the system.

Gabriel Rivière (Université Lille 1)
Eigenmodes of the wave equation, chaotic dynamical systems and entropy.

This talk is designed for a general audience. We will discuss some properties of stationary solutions of the wave equation on a bounded euclidean domain (or more generally on a compact manifold). In other words, we will be concerned with the properties of the eigenfunctions of the Laplacian.

Our main focus will be on domains for which the "free motion" of a particle (the geodesic flow) enjoys chaotic properties. We will explain how this chaotic feature influences the properties of the eigenfunctions in the high frequency limit. Finally, we will describe some recent progresses that have been obtained on these questions thanks to the use of the so-called Kolmogorov-Sinai entropy.

Nessim Sibony (Université Paris Sud)
Entropy in holomorphic dynamics.

This talk is designed for a general audience. The entropy of a dynamical system is a number measuring how much the system is chaotic. More precisely it measures the growth of the number of orbits one can distinguish before time n, at every scale. It is an important invariant at the center of the theory of dynamical systems.One of the goals is to construct an invariant measure on the most chaotic part, to study its geometry, and the rates of expansion-contraction (Lyapounov-exponents). The abstract theory is well estabished, Oseledec, Pesin... To decide wether it applies in concrete examples is another story, most of the time out of reach.

I will first recall Bowen's definition of entropy. Then I will discuss the notion in the context of holomorphic or meromorphic maps between compact Khler manifolds. In particular it's relation with dynamical degrees, which describe the average expansion. This is strongly related to works by Gromov and Yomdin. Complex dynamics is a remarkable case where entropies can be computed explicitly in many situations. Measures of maximal entropy can be constructed, although the systems are not uniformly hyperbolic in general.

I will also develop a notion of entropy for laminations by hyperbolic Riemann surfaces. In that case, we use the hyperbolic time. The finiteness of entropy is related to the transverse Hölder continuity of the Poincaré metric. The theory can be extended to cover the case of singular foliations on complex surfaces. The lecture is based on Joint works with T.C Dinh and V.A. Nguyen.

Karl-Theodor Sturm (Rheinische Friedrich-Wilhelms-Universität Bonn)
Ricci Curvature and Entropy

We give a brief survey on Ricci curvature bounds and heat flow on metric measure spaces. Generalized bounds for the lower Ricci curvature on metric measure spaces $(X; d; m)$ have been introduced by Lott & Villani and the author in terms of convexity properties of the relative entropy regarded as function on the Wasserstein space on the given space X. Besides Riemannian manifolds, the class of examples include Alexandrov spaces, Finsler spaces, path spaces as well as limits and constructions of such spaces (Euclidean cones, warped products). It turned out that - in great generality - the heat flow on metric measure spaces $(X; d; m)$ can be defined equivalently as gradient flow on $L^2(X)$ for the energy or as gradient flow on the L^2-Wasserstein space $P^2(X)$ for the relative entropy. We will present recent results on the equivalence of the "entropic curvature-dimension condition" and Bochner's inequality. This also includes sharp gradient estimates and new contraction properties of the heat flows in L^2-Wasserstein metric. Among the applications are Li-Yau estimates on metric measure spaces satisfying a curvature-dimension condition.

Constantin Vernicos (Université Montpellier 2)
Volume entropy of Hilbert geometries and approximability of convex bodies

3 Computer and Internet

- Two computers are in self-service in the coffee room of IRMA.
- There is an Internet access datas for laptops:

wireless network: osiris **login**: conf-entropy **password**: mathphys92

4 City plan and list of participants

- See the back of the program.
- There are a lot a small restaurants around IRMA, especially at place St-Nicolas-aux-ondes, rue des balayeurs, rue de l'abreuvoir, rue de Zürich, rue de la Krutenau.
- A standard place for the lunch is the university cafeteria (RU on the map).
- There are ATMs at "place des foins" or "place de Zürich".

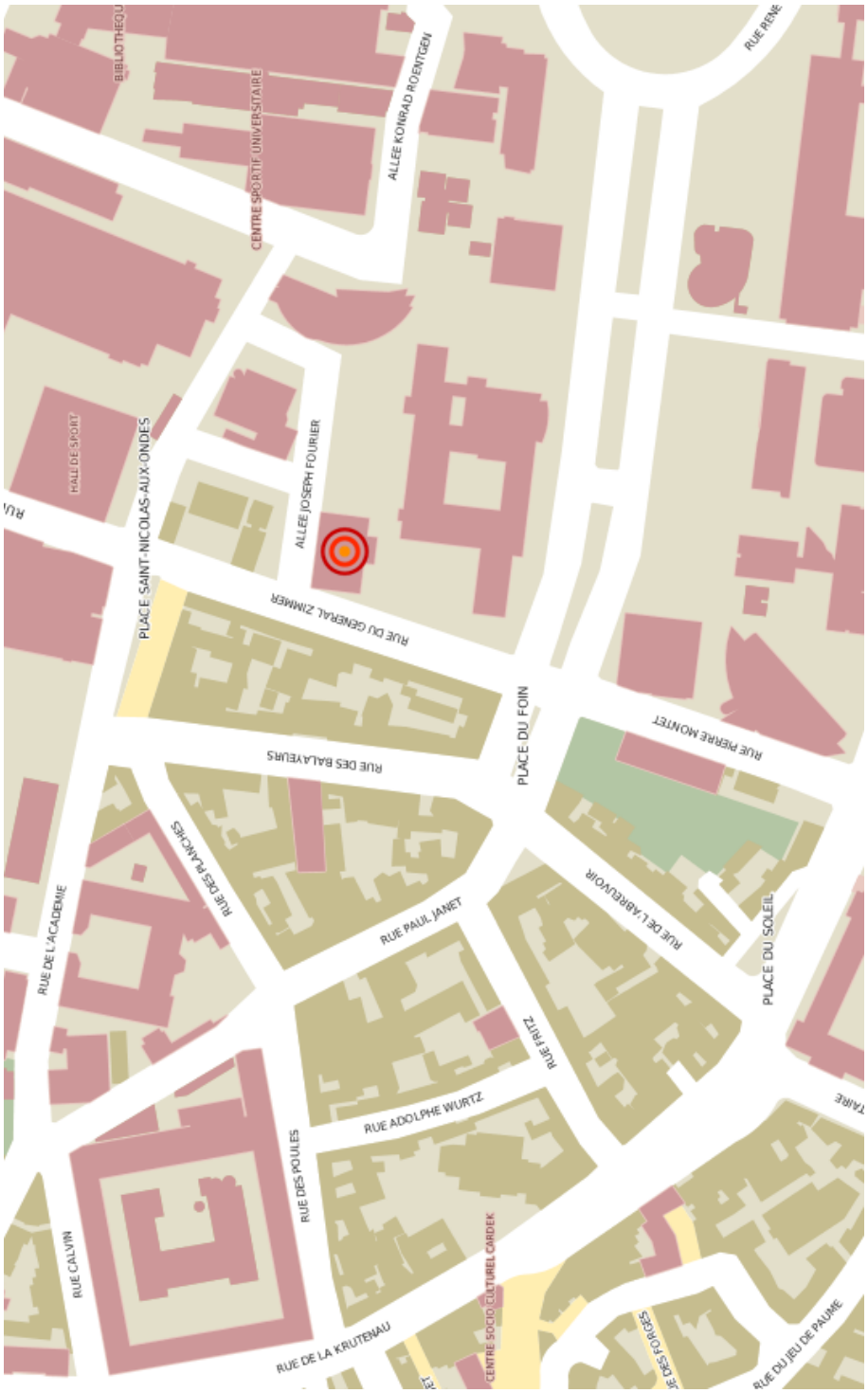

Figure 1: Around IRMA

Alberge, Vincent	Université de Strasbourg
Angst, Jürgen	Université Rennes 1
Annafi, Tajudeen Amadu	University of Ghana
Badr, Ramy	Universität Leipzig
Barbaresco, Frédéric	THALES
Benrabia, Djaafer	USTHB Alger
Bérard, Jean	Université de Strasbourg
Besson, Gérard	Université Grenoble 1
Bouchet Freddy	ENS Lyon
Boughazi, Hichem	EPSECG Tlemcen
Caddeo, Renzo	Università di Cagliari
Carfora, Mauro	Università di Pavia
Chafaï, Djalil	Université Paris-Dauphine
Charalampos, Charitos	Agr. University of Athen
De Gérando, Stéphane	Paris
Degerli, Bahar	Yeditepe University
Fathi, Max	Université Paris 6
Franchi, Jacques	Université de Strasbourg
Hijazi Oussama	Université de Lorraine
Hoang duc, Auguste	Université de Strasbourg
Iddaoudi, Abdellah	Université Ibn Zohr – Agadir
Jakubsky, Vit	Acad. of Science Czech Rep.
Juillet, Nicolas	Université de Strasbourg
Karlsson, Anders	Université de Genève
Korkmaz, Mustafa	METU Ankara
Kucuck, Kubra	Yeditepe University
Maes, Christian	KU Leuven
Papadopoulos, Athanase	Université de Strasbourg
Pastant, Nicolas	Colmar
Perez, Asher	Université de Strasbourg
Peyre, Rémi	Université de Lorraine
Pfister, Charles-Édouard	EPF Lausanne
Rivière, Gabriel	Université Lille 1
Rodado Amaris, Armando	Massey University
Sibony, Nessim	Université Paris 11
Sikandar, Arslan	LUMS
Sturm, Karl-Theodor	Universität Bonn
Tahri, Kamel	EPSECG Tlemcen
Tardif, Camille	Université Paris 6
Vernicos, Constantin	Université Montpellier 2
Zanardo, Paolo	Università di Padova