Verification of Gyrokinetic codes: Theoretical background & Numerical implementations

NumKin 2016, IRMA, Strasbourg, France

1 NMPP, Max Planck Institute für Plasmaphysik
2 TOK, Max Planck Institute für Plasmaphysik
Enabling Research Project on Verification of Gyrokinetic codes

- **Germany**
  Max Planck Insitute for Plasma Physics (Garching and Greifswald)
- **Switzerland**
  SPC-EPFL, Lausanne
- **Finland**
  Aalto University
- **Great Britain**
  University of Warwick
- **France**
  CEA Cadarache,
  Université Paris IV, Rennes, Toulouse, Lorraine, Bretagne
  IRMA, Maison de la Simulation (Saclay)
- **USA**
  Saint Michel’s College, VT
VeriGyro Project @ IPP Garching

Interdisciplinary Project in IPP Max Planck (Garching)

NMPP Division (Numerical Methods for Plasma Physics)
• Development and implementation of new algorithms for fusion and astrophysical plasma modeling
• Development of libraries (i.e. Selalib)
• Verification of existing codes
• Participants: N. Tronko, E. Sonnendrücker

TOK (Tokamak Theory)
• Development of major codes
• Plasma modeling and comparison with experimental results (ASDEX Upgrade)
• Participants: T. Görl, A. Bottino, B. D. Scott

Enabling Research Project VeriGyro

NumKin 2016
VeriGyro Project: Motivation

• Verification of Global (Electromagnetic) Gyrokinetic codes: Why?

• Most popular tools for magnetised plasmas simulations:
  
  • Significant Development since last 10 years
  • Electrostatic gyrokinetic implementations: well established
    
  
  • Global Electromagnetic gyrokinetic implementations:
    
    • Another level of complexity with respect to electrostatic codes
    • A lot of freedom for approximations (Poisson and Ampère equations)
    • Different codes use different version of GK equations

• What was missing in the beginning of VeriGyro (2014) ?

  • General hierarchy of the GK equations implemented numerically
  • Global Electromagnetic Intercode benchmark
VeriGyro: Strategy & Accomplishments

- **Theory**: Establishing Hierarchy of models for existing codes
  *(Task leader Natalia Tronko)*

  - Systematic derivation from the Variational GK framework
    *[Tronko, Bottino, Sonnendrücker, Phys. Plasmas 2016]*

  - Verification of approximations consistency and regimes of applicability

- **Intercode Benchmark**: challenging electromagnetic problems
  *(Task leader Tobias Görler)*

  - Implicit numerical schemes verification
  - Microinstability characterization and exploration of required grid
  - Microturbulence: one of the next steps

VeriGyro: Intercode Benchmark Participating codes

- PIC codes:
  - EUTERPE \([\text{Kornilov Phys. Pl. 2004}]\) R. Kleiber (IPP Greifswald)

- Eulerian codes:

- Semi-Lagrangian:
VeriGyro: Numerical resources

- Supercomputer Helios (Japan) Hydra (IPP Garching)
- Since Oktober 2017 Marconi Fusion (Italy)
- Using between 512 and 2048 processors (*for ORB5*)
- CPU time pro simulation: 2h (adiab) towards several days (electromagnetic linear)
PART I: Theoretical foundations

- **Gyrokinetic theory for numerical implementations:**
  - Systematic derivation from the first principle of dynamics

  - Second order Gyrokinetic Maxwell-Vlasov Parent model
  - **Focus on the ORB5** code model

- **Continuous descriptions**
  - Gyrokinetic dynamical reduction on **single** particle phase space
  - Systematic **coupling** of reduced particle dynamics with fields dynamics
  - **Conservation laws** and code diagnostics (**ORB5**)  

- **Discretized description (ORB5)**
  - Finite Element Monte-Carlo Particle-In-Cell Lagrangian
  - Discrete equations of motion
• Establishing common test case:
  • Adiabatic electrons simulations
  • Electrostatic (2 species)
  • Electromagnetic linear Benchmark

• Identifying differences in simulations:
  in function of the implemented model and numerical scheme
GK Orderings

- Guiding-center: background quantities
  \[ |\rho_i \nabla \ln B| \sim \epsilon_B \]

- Gyrocenter: fluctuating fields
  \[ \left| \frac{\delta f}{F} \right| \sim \frac{c |\delta E_\perp|}{B v_{th}} \sim \frac{|\delta B|}{B} \sim \epsilon_\delta \]

- Gyrokinetics
  \[ k_\perp \rho_i \sim 1 \]

- Maximal ordering
  \[ \epsilon_B \sim \epsilon_\delta \]

- Drift-kinetics
  \[ k_\perp \rho_i \ll 1 \]

- Code ordering
  \[ \epsilon_B \ll \epsilon_\delta \]

[Brizard, Hahm Rev.Mod.Phys., 2007]
Local Particle coordinates

- Non-canonical local particle coordinates 6D:
  \[ Z^\alpha = (x, v) \rightarrow (x, v_\parallel, \mu, \theta) \]

- Rotating particle basis:
  \[
  \begin{align*}
  \hat{\perp} &= -\hat{b}_1 \sin \theta - \hat{b}_2 \cos \theta \\
  \hat{\rho} &= \hat{b}_1 \cos \theta - \hat{b}_2 \sin \theta
  \end{align*}
  \]

- Magnetic Momentum: adiabatic invariant
  \[ \mu = \frac{mv_\perp^2}{2B} \]

- Velocity vector decomposition
  \[ v = v_\parallel \hat{b} = v_\perp = v_\parallel \hat{b} + (2\mu mB)^{1/2} \hat{\perp} \]

- Goal of the dynamical reduction: build up a change of variables such that \( \dot{\mu} = 0 \) fast gyromotion is uncoupled

- Relevant dynamics on the reduced (4+1)D phase space: \( (X, v_\parallel; \mu) \)
Two step dynamical reduction

• Two polarization displacements \( x \equiv X + \rho_0 (X, \mu, \theta) + \rho_1 (X, \mu, \theta) \)

• Guiding-center displacement

\[
\rho_0 \equiv \frac{mc}{e} \sqrt{\frac{2\mu}{mB}} \hat{\rho} \equiv \rho_0 \hat{\rho} \\
\sim \mathcal{O}(\epsilon_B^0)
\]

• Gyrocenter displacement

\[
\rho_1 = \frac{mc^2}{B^2} \nabla_\perp \left( \phi_1 (X) - \frac{p_z}{mc} A_{1\parallel}(X) \right) \\
\sim \mathcal{O}(\epsilon_\delta)
\]

• First order electromagnetic potential

\[
\psi_1 = \phi_1 - \frac{e}{mc} p_z A_{1\parallel}
\]

• Canonical gyrocenter momentum:

\[
p_z = m v_\parallel + \frac{e}{c} A_{1\parallel} (X + \rho_0 + \rho_1)
\]
Second order Gyrocenter Lagrangian

- Particle Lagrangian on the reduced phase space \((X, p_z, \mu, \theta)\)

\[ L_p = \left( \frac{e}{c} A + p_z \hat{b} \right) \cdot \dot{X} + \frac{mc}{e} \mu \dot{\theta} - H \]

- Non-perturbed dynamics (guiding-center)

\[ H_0 = \frac{p_z^2}{2m} + \mu B \]

- First order dynamics

\[ H_1 = eJ_0^{gc} (\phi_1 (X + \rho_0)) - \frac{ep_z}{mc} J_0^{gc} (A_1 (X + \rho_0)) \]

The gyroaveraging operator: with respect to the guiding-center displacement

\[ J_0^{gc} [\psi_1 (X + \rho_0(\theta))] = \frac{1}{2\pi} \int_0^{2\pi} d\theta \, \psi_1 (X + \rho_0(\theta)) \]
Second order Hamiltonian hierarchy

- **Full FLR 2\textsuperscript{nd} order electromagnetic** Hamiltonian equivalent to Hahm 1988 electrostatic model

  \[
  H_2^{\text{full}} = \frac{1}{2m} \left( \frac{e}{c} \right)^2 J_0^{gc} (\psi_1 (X + \rho_0)) - \frac{e^2}{2mc} J_0^{gc} \left( \frac{\partial}{\partial \mu} \tilde{\psi}_1 (X + \rho_0) \tilde{\psi}_1 (X + \rho_0) \right)
  \]

  Electromagnetic coupling between GK Poisson and Ampère equations

- **Truncated**: All 2\textsuperscript{nd} FLR terms

  \[
  H_2^{\text{FLR}} = \frac{e^2}{2mc^2} A_1 (X)^2 + \frac{\mu}{2B} \left\| \nabla \perp A_1 (X) \right\|^2 + \frac{1}{2} \frac{\mu}{B} A_1 (X) \nabla^2 \perp A_1 (X)
  \]

  \[
  - \frac{mc^2}{2B^2} \left\| \nabla \perp \phi_1 (X) - \frac{p_z}{mc} A_1 (X) \right\|^2
  \]

- **ORB5 2\textsuperscript{nd} Order**: Second order FLR magnetic part and first order FLR in Electrostatic Polarization part

  \[
  H_2^{\text{ORB5}} = \frac{e^2}{2mc^2} A_1 (X)^2 + \frac{1}{2} \frac{\mu}{B} A_1 (X) \nabla^2 \perp A_1 (X) - \frac{mc^2}{2B^2} \left\| \nabla \perp \phi_1 (X) \right\|^2
  \]

  Uncoupled GK Poisson and Ampère equations

NumKin 2016
Gyrokinetic field theory

Which components we do really need for building up a self-consistent model for a code? [Sugama Phys. Pl. 2000, Brizard PRL 2000]

\[
L = \sum_{\text{sp}} \int dV \ dW \ F(Z_0, t_0) \ L_p \left( Z[Z_0, t_0; t], \dot{Z}[Z_0, t_0; t]; t \right) + \int dV \frac{|E_1|^2 - |B_{1\perp}|^2}{8\pi}
\]

particles

\[
Z = (X, p_z, \mu, \theta); \ dW = \frac{2\pi}{m^2} B^* d\rho_z d\mu; \ B_{1\perp} = \left| \nabla_{\perp} A_{1||} \right|^2
\]

fields

Goal: Coupling reduced particle dynamics with fields within the common mathematical structure

Getting consistently reduced set of Maxwell-Vlasov equations

\[
F(Z_0, t_0) \quad \text{Distribution function of species “sp” at arbitrary initial time} \quad t_0
\]

\[
L_p \quad \text{Gyrocenter Lagrangian: reduced motion of a single particle}
\]
How to build up a gyrokinetic model consistently?

- All approximations **should** be performed on the Lagrangian $L$ **before** deriving the equations of motion

- Approach guarantees energetic consistency of final equations

- Symmetry properties of Lagrangian are automatically transferred to the equations

- **Set up polarization effects into the system**
  (hierarchy of models):

  - Choice of the second order gyrocenter Hamiltonian defines reduced field-particles dynamics
Quasi-neutrality approximation

- Electrostatic contributions from fields and gyrocenter polarization

\[
\int dV \frac{|E_1|^2}{8\pi} + \int dW \: dV \: F \frac{mc^2}{2B^2} |\nabla \perp \phi_1|^2 = \frac{1}{8\pi} \int dV \left( 1 + \frac{\rho_s^2}{\lambda_d^2} \right) |\nabla \perp \phi_1|^2
\]

Polarization from the second order particle dynamics: ORB5 model

\[
H_{ORB5}^2 = \frac{e^2}{2mc^2} A_{1\parallel}^2 (X) + \frac{1}{2} \frac{\mu}{B} A_{1\parallel} (X) \nabla \perp^2 A_{1\parallel} (X) - \frac{mc^2}{2B^2} |\nabla \perp \phi_1 (X)|^2
\]

\[
\lambda_d^2 = \frac{k_B T_e}{4\pi n e^2}
\]

Debye length

\[
\rho_s^2 = \frac{k_B T_e mc^2}{e^2 B^2}
\]

Sound ion Larmor radius

\[
\frac{\rho_s^2}{\lambda_d^2} = \frac{4\pi n mc^2}{B^2} = \frac{c^2}{v_A^2} \gg 1
\]
Linearised polarisation approximation

\[ L = \sum_{sp} \int dW \, dV \left( \left( \frac{e}{c} \mathbf{A} + p_z \mathbf{b} \right) \cdot \dot{\mathbf{X}} + \frac{mc}{e} \mu \dot{\theta} - H \right) F - \int dV \frac{B_{\perp}^2}{8\pi} \]

Up to \( \sim \mathcal{O}(\epsilon_\delta^2) \)

- Initially: Hamiltonian decomposition \( H = H_0 + H_1 + H_2 \)
- Dynamical part of distribution function \( (H_0 + H_1)F \)
- Background part of distribution function \( H_2F_0 \)

This approximation leads to linearised field equations

NumKin 2016
GK Field equations

• Polarization equation in a weak form

\[ \epsilon \delta \sum_{\text{sp}} \int d\Omega \, F_0 \frac{mc^2}{B^2} \left| \nabla \perp \phi_1 \right|^2 = \sum_{\text{sp}} \int d\Omega \, J^{gc}_0 (\phi_1) (F_0 + \epsilon \delta F_1) \]

• Ampere’s equation in a weak form

\[ \frac{\delta L}{\delta A_{1\parallel}} \circ \hat{A}_{1\parallel} = 0 \]

Test function

\[ \hat{A}_{1\parallel} = A_{1\parallel} \]

\[ \int dV \frac{1}{4\pi} \left| \nabla \perp A_{1\parallel} \right|^2 + \epsilon \delta \sum_{\text{sp}} \int d\Omega \frac{e^2}{mc^2} \left( J^{gc}_0 A_{1\parallel} \right)^2 F_0 = \sum_{\text{sp}} \int d\Omega \frac{p_z}{mc} J^{gc}_0 (A_{1\parallel}) (F_0 + \epsilon \delta F_1) \]

Weak form of the equations of motion is suitable for the finite elements discretisation!!!!!
Vlasov equation

Vlasov equation is reconstructed from the characteristics

$$\frac{\delta L}{\delta \mathbf{Z}} = 0$$

$$\mathbf{Z} = (\mathbf{X}, p_z, \mu)$$

$$\mathbf{X} = \frac{\partial (H_0 + H_1)}{\partial p_z} \frac{\mathbf{B}^*}{B^*_\parallel} - \frac{c}{eB^*_\parallel} \hat{\mathbf{b}} \times \nabla (H_0 + H_1)$$

$$\dot{p}_z = -\frac{\mathbf{B}^*}{B^*_\parallel} \cdot \nabla (H_0 + H_1)$$

$$\frac{d}{dt} f(\mathbf{Z} [\mathbf{Z}_0, t_0, t] ; t) = \frac{\partial}{\partial t} f(\mathbf{Z}, t) + \frac{d\mathbf{Z}}{dt} \cdot \frac{\partial}{\partial \mathbf{Z}} f(\mathbf{Z}, t)$$

**Linearized characteristics: only** $H_0$ **contributions**
PIC code ORB5 “J.E” diagnostics

PIC code: separation in calculation of fields and particles dynamics:

**Particles:** characteristics     **Fields:** Grid evaluation

Controlling quality of the simulation: contributions from particles energy and fields energy should be calculated independently.

Calculation of the instability growth rate via two different methods

\[
\gamma = \frac{1}{2E_F} \frac{dE_k}{dt} = - \frac{1}{2E_F} \frac{dE_F}{dt}
\]

- \( \frac{dE_k}{dt} \) Calculation from the linear characteristics using the Noether method for energy conservation
- \( \frac{dE_F}{dt} \) Direct evaluation on the grid at each time step

J.E  Diagnostics
ORB5 diagnostics derivation: main steps

1. Expression for second order energy density

\[ \mathcal{E}^{\text{ORB5}} = \sum_{\text{sp}} \int dW \, dV \, H_0 \, (F_0 + \epsilon_\delta F_1) + \sum_{\text{sp}} \int dW \, dV \, H_1 \, (F_0 + \epsilon_\delta F_1) \]

\[ + \sum_{\text{sp}} \int dW \, dV \, H_2^{\text{ORB5}} \, F_0 + \int dV \, \frac{|\nabla_\perp A_{1\parallel}|^2}{8\pi} \]

2. Use Equations of motion to simplify

\[ \mathcal{E}^{\text{ORB5}} = \sum_{\text{sp}} \int dV \, dW \, \left( H_0 - \epsilon_\delta \frac{e}{m} \mathcal{J}_0^g \left( A_{1\parallel} \right) \right) \, (F_0 + \epsilon_\delta F_1) + \sum_{\text{sp}} \int dV \, \frac{|\nabla_\perp A_{1\parallel}|^2}{8\pi} \]

\[ + \sum_{\text{sp}} \int dV \, dW \, \left( \frac{e^2}{2mc^2} A_{1\parallel}^2 + \frac{mc^2}{2B^2} |\nabla_\perp \phi_1|^2 \right) \, F_0 \equiv \mathcal{E}_k + \mathcal{E}_F \]

3. Separate kinetic and field parts. Use power Balance equation for simulations diagnostics

\[ \gamma = \frac{1}{2\mathcal{E}_F} \frac{d\mathcal{E}_k}{dt} = -\frac{1}{2\mathcal{E}_F} \frac{d\mathcal{E}_F}{dt} \]
Electromagnetic “J.E”

\[ \gamma = \frac{1}{2\mathcal{E}_F} \sum_{sp} \int dV \, dW \, (F_0 + \epsilon \delta F_1) \left[ e \nabla \mathcal{J}_{0g}^{ge} (\psi_{1ge}) \cdot \dot{X} \bigg|_0 - \frac{1}{c} \mathcal{J}_{0g}^{ge} (A_{1||ge}) \left( \frac{\dot{p}_z}{m} \right) \bigg|_0 \right] \]

\[ \frac{d\mathcal{E}_k}{dt} \]

\[ (X|_0, p_z|_0) \quad \text{Linearized characteristics} \]

Energy diagnostics does not require knowledge of full particles trajectories

Diagnostics of linear simulations

Detailed diagnostics: identification of instabilities stabilizing and destabilizing mechanisms (Part II)
PART I: Conclusions

Continuous Dynamics:

• GK dynamical reduction starts with reduced particle Lagrangian
• Reduced particle dynamics defines hierarchy of GK Maxwell-Vlasov models

• Common mathematical structure for coupling reduced particle dynamics with those of electromagnetic fields

• Derivation of simulation diagnostics via conservation laws, exactly corresponding to the fields – gyrocenters Lagrangian on the infinite dimensional phase space: *key element for models verification*

• **ORB5 model: inside the hierarchy**

• **Perspective: include GENE code: hierarchy model & diagnostics**
First principle dynamics discretisation:

- Monte-Carlo Lagrangian
- Discrete equations of motion
PIC Code Cycle

Start here:
Initializing particle initial conditions

**Charge assignment:**
calculation of fields value at the particles positions

\[(E, B) \rightarrow (\dot{X} = \ldots; \dot{p}_|| = \ldots)\]

**Charge & current deposition:**
calculation of current and charge on grid: source terms of field equations

**Integrating GK fields equations:**
fields solver: finite elements

\[(E, B)_i \leftarrow J_i\]

**Integrating particle’s GK characteristics:**
particles pushing

\[\dot{X} = \ldots; \dot{p}_|| = \ldots;\]
Discrete Gyrokinetic field theory

- Monte-Carlo Lagrangian density

\[
\mathcal{L} = \sum_{sp} \int d\mathbf{Z} \ F_{sp} (\mathbf{Z}, t) \ L_p \left( \mathbf{Z}, \mathbf{\dot{Z}} \right) + L_F
\]

\[
\mathcal{L}_{MC} = \sum_{sp} \frac{1}{N_p} \sum_{k=1}^{N_p} F_{sp} (\mathbf{Z}_k(t), t) \ L_{sp} \left( \mathbf{Z}_k(t) \mathbf{\dot{Z}}_k(t) \right) + L_F
\]

Systematic derivation of discretized equations

- Field discretization: \textit{finite-element method}

\[
\Psi_h (\mathbf{x}, t) = \sum_{\mu=1}^{N_g} \Psi_{\mu}(t) \Lambda_\mu (\mathbf{x})
\]

Cubic splines

\[
\Lambda_\mu (\mathbf{x}) = \Lambda_j (x_1) \Lambda_k (x_2) \Lambda_l (x_3)
\]

- \textit{Galerkin approximation}
- Variations on finite-dimensional phase-space
Discrete Poisson and Ampere equations

- Discretization of particle’s density using importance sampling: this is why \( w_k \) are not simply equal to one

\[
n_{gc} = \int dW f_N \simeq \sum_{k=1}^{N} \frac{2\pi B^*_{||} (X_k)}{m^2} w_k \delta (X - X_k(t))
\]

- Discretized quasineutrality equation

\[
\frac{\partial L_{MC}}{\partial \phi_1} = 0
\]

\[
\sum_{\mu=1}^{N_g} \Phi_{\mu} \sum_{sp} \int d\Omega \frac{mc^2}{B^2} \nabla_{\perp} \Lambda_{\nu} \cdot \nabla_{\perp} \Lambda_{\mu} = \sum_{sp} \frac{1}{N_p} \sum_{k=1}^{N_p} w_k (eJ_{0,k} \Lambda_{\nu} (X_k))
\]

- Discretized Ampère’s equation

\[
\frac{\partial L_{MC}}{\partial A_{1||}} = 0
\]

\[
\sum_{\mu=1}^{N_g} A_{1||\mu} \left( \frac{1}{4\pi} \int dV \nabla_{\perp} \Lambda_{\nu} \cdot \nabla_{\perp} \Lambda_{\mu} - \sum_{sp} \int d\Omega \frac{e^2}{mc} J_0 \Lambda_{\nu} \cdot J_0 \Lambda_{\mu} \right) = \sum_{sp} \frac{1}{N_p} \sum_{k=1}^{N_p} w_k (eJ_{0,k} \Lambda_{\nu} (X_k))
\]
PART II: Models approximations: open questions

• Deriving discretised reduced model from Lagrangian framework:

  • Weak form of equations issued from first principle: implementable in the PIC code

  • Long wavelength approximation can be obtained inside the Lagrangian framework

• **Open question**: all the approximations can be derived from the Lagrangian?
PART II: Linear Intercode Benchmark

- **Common framework:** CYCLONE Base Test Case

- **Studies of ITG/KBM transition:**
  - Linear Electromagnetic Beta-scan
  - Toroidal modes (“n-scan”) with nominal beta (c.a.1%)
    


- **Focus on the ORB5:** identification of stabilizing/destabilizing mechanisms with electromagnetic J.E diagnostics
CYCLONE Base Case

- Common framework for benchmark: [Dimits, Phys. Pl. 2000]
- The original discharge DIII-D: electrostatic simulations, adiabatic electrons
  H- mode shot #81499 at t=4000 ms; flux tube label r=0.5a

\[ q(r) = 2.52 (r/a)^2 - 0.16 (r/a) + 0.86 \]

- Temperature and density profiles
  \[ A = (n, T) \]
  \[ A/A(r_0) = \exp \left[ -\kappa_A \, w_A \frac{a}{L_{\text{ref}}} \tanh \left( \frac{r-r_0}{w_A a} \right) \right] \]
  \[ L_{\text{ref}}/L_A = -L_{\text{ref}} \partial_r \ln A(r) = \kappa_A \cosh^{-2} \left( \frac{r-r_0}{w_A a} \right) \]

\[ L_{\text{ref}} = R_0 \]
\[ \kappa_A \text{ characteristic width} \]
\[ w_A \text{ maximal amplitude} \]
Linear electromagnetic Beta scan

- Looking at one of the most instable modes $n = 19$
- Successful comparison of 4 different codes (2xPIC and 2xEulerian)
- All codes agree at the ITG/KBM transition
- Threshold shifted comparing to flux-tube growth rate

**Important for experiment**

Growth Rate scan

**ORB5 code**
$n_{\text{tot}} = 8 \times 10^6$
$n_{\text{tot}} = 16 \times 10^6$
Looking at one of the most unstable modes \( n = 19 \)

**Frequencies scan**

**ORB5 code**

\[ n_{\text{tot}}^{\text{De}} = 8 \times 10^6 \]
\[ n_{\text{tot}}^{e} = 16 \times 10^6 \]
**Toroidal mode scan at nominal beta**

**GENE/GKW:** transition towards Trapped Electron Modes at n=35

**ORB5:** Instability growth rate breaks down at large values of n>35 because the *drift-kinetic* approach is outranged \( k_\theta \rho_s = 0.54; \)

\[ k_\theta \rho \ll 1 \]

\[ n = 19 \]
Toroidal mode scan at nominal beta

**GENE/GKW:** solving the Poisson equations with full FLR corrections

**ORB5:** long-wavelength approximation solver

\[ n = 19 \]
Toroidal mode scan: electrostatic

- **GENE/GKW:** transition towards Trapped Electron Modes at \( n = 35 \)

- **ORB5:** Instability growth rate breaks down at large values of \( n > 35 \) because the *drift-kinetic* approach is outranged \( k_\theta \rho_s = 0.54 \);

- **ORB5:** full FLR Poisson equation solver implementation: 
  
  *J. Dominski PhD thesis (SPC, EPFL)*
Radial structure of modes: ORB5/GENE

- **Novelty in Benchmark**
- Fine structure in electrostatic potential:
- Mode-Rational Surfaces (MRS): electrons can not be adiabatic
- *Costly simulations: demanding high radial resolution*

[Dominski, Phys. Pl. 2015]

- **ORB5**: MRS are smoothed because of the long-wavelength solver implementation
Radial structure of modes: electrostatic potential

GENE Global

\( n = 25 \)

ORB5
Radial structure of modes: electromagnetic potential

$G E N E \text{ Global} \quad n = 25 \quad O R B 5$
ITG/KBM Bifurcation study with ORB 5

- **Use J.E diagnostics derived from the energy conservation:**
  detailed balance of stabilizing/destabilizing mechanisms

- **Low beta magnetic ITG**
  \[ \beta = 0.025\% \]
- “Gradient-driven” \( \sim \nabla B \)
- Stabilizing magnetic effects

- **Increasing magnetic** \( \beta = 0.25\%: \)
  - Driven by
    \[ p_z = p_{\parallel} + \frac{e}{c} A_{1\parallel} \]
    and \( \sim A_{1\parallel} \)

Mostly destabilized by magnetic effects
- **Stabilizing drift effects**
  \( \sim \nabla B \)
• **Bifurcation between ITG and KBM:**

  • Mode beating

  \[ \beta = 0.68\% \]

• **KBM instability:**

  \[ \beta = 0.875\% \]

  • Drift-stabilising
  • Driven by electromagnetic mechanisms

  \[ p_z = p \parallel + \frac{e}{c} A_{1\parallel} \]

  \[ \sim A_{1\parallel} \]
Conclusions and perspectives

• **Achievements**
  • *Linear* electromagnetic benchmark campaign: toroidal modes scan for nominal $\beta=1\%$ value
  • Expensive simulations (especially at higher toroidal numbers):
    • High radial resolution
    • High number of particles (markers)
    • Small time step
  • Building up hierarchy of GK models for existing codes
    • General second order Maxwell-Vlasov model
    • Electromagnetic Lagrangian formulation ORB5

• **Perspectives**
  • Detailed comparison between *nonlinear* electromagnetic simulations between Lagrangian (ORB5) and Eulerian (GENE) Challenging!!!!

[Goerler, Tronko, Hornsby et al, PoP 2016]

[Tronko, Bottino, Sonnendrücker, PoP 2016]
References

• **Numerics**
  - Bottino, A., Vernay, T., Bruce, S., PPCF, **53**, 124027, 2011

• **Theory**
Outline (passer du temps)

- Verification of Global (Electromagnetic) Gyrokinetic codes (VeriGyro):
  - Electromagnetic: another level of complexity, a lot freedom for approximations (Ampere equation) we notice that different codes uses different version of GK equations) This is why we want to have a common verification being done.
- Theoretical line
- At the same time: Benchmark the code on challenging electromagnetic problems micro-turbulence and (MHD modes are also part of codes) it is not a stupid Benchmark at all... Why we need theory: each code slightly different theory.
  
- To my knowledge it was the only GK enabling research project accepted
  - Most popular tools for plasma turbulence investigation
  - Extended development over last 10 years
  - Variety of implemented models

- Building up hierarchy of Electromagnetic Gyrokinetic models implemented into the codes (task leader N.Tronko):
  - Goal: understand the limitations of existing codes
    - Systematic derivation from the Variational GK framework
    - Verification of approximations consistency
    - Identification of regimes of applicability

- Multi-code Benchmark: implicit numerical schemes verification (task leader T.Goerler)
  - Establishing a common test-case framework
\[
\gamma = \frac{1}{2\mathcal{E}_F} \sum_{sp} \int dV \ dW \ (F_0 + \epsilon_\delta F_1) \nabla \mathcal{J}_0^{gc} (\psi_1) \left( v_\parallel + v_{\nabla P} + v_{\nabla B} \right)
\]

\[
- \frac{1}{2\mathcal{E}_F} \sum_{sp} \int dV \ dW \ (F_0 + \epsilon_\delta F_1) \nabla \mathcal{J}_0^{gc} (A_{1\parallel}) \left( \mu B \nabla \cdot \hat{b} + \frac{\mu c}{e B^*_\parallel} p_z \hat{b} \times \left( \hat{b} \times \frac{\nabla \times B}{B} \right) \cdot \nabla B \right)
\]

- Drift velocity components
- Useful for analyzing mechanisms behind instabilities
Convergence studies: number of markers

Dependencies on the number of markers in the system for relative error

\[ \text{RelErr} = \frac{J.E + E_{\text{transf}}}{E_{\text{transf}}} \]

Mean value of RelErr

\[ m = \frac{1}{N} \sum_{i=1}^{N} x_i \]

Variance value of RelErr

\[ V(X) = \frac{1}{N} \sum_{i=1}^{N} (x_i - m)^2 \]

Calculation from the characteristics

\[ J.E \equiv \frac{1}{E_F} \frac{dE_k}{dt} \]

Direct calculation from finite elements method

\[ E_{\text{transf}} \equiv \frac{1}{E_F} \frac{dE_F}{dt} \]

NumKin 2016
Number of markers convergence: RelErr

\[ n_{\text{ptot}_e} = 64 \times 10^6 \quad n_{\text{ptot}_{De}} = 8 \times 10^6 \quad n_{\text{ptot}_e} = 16 \times 10^6 \]

n_{\text{ptot}_e} = 4 \times 10^6

How number of markers affects noise level in the simulation
Time step convergence

Separation between energy diagnostics with increasing the time step
From general second order theory to ORB5

- Quansineutrality condition
- Uncoupled Quasineutrality and Ampère equations

- Reduced particle dynamics simplification via $H_2$
  - Long-wavelength approximation
  - Purely electrostatic polarization
\[ H^\text{full}_2 = \frac{1}{2m} \left( \frac{e}{c} \right)^2 \langle \psi_1 (\mathbf{X} + \rho_0) \rangle - \frac{e^2}{2mc} \left< \frac{\partial}{\partial \mu} \tilde{\psi}_1 (\mathbf{X} + \rho_0) \tilde{\psi}_1 (\mathbf{X} + \rho_0) \right> \]