Résumés des exposés

Squares in arithmetic progression.

It is an easy exercise to find examples of integer squares in arithmetic progression of length three. What about four term arithmetic progressions of integer or rational squares? It is well-known that squares in arithmetic progression are related to certain elliptic curves and the famous congruent number problem, respectively. We report about investigations on this topic over number fields.

On Newman polynomials which divide no Littlewood polynomial.

Recall that a polynomial $P(x) \in \mathbf{Z}[x]$ with coefficients in $\{0, 1\}$ and constant term 1 is called a Newman polynomial, whereas a polynomial with coefficients $\{-1, 1\}$ is called a Littlewood polynomial. Is there an algebraic number which is a root of some Newman polynomial but is not a root of any Littlewood polynomial? In other words (but not equivalently), is there a Newman polynomial which divides no Littlewood polynomial? In a joint paper with A. Dubickas, for each Newman polynomial P of degree at most 8, we found a Littlewood polynomial divisible by P. Moreover, we have shown that every trinomial of the form $1+ux^a+vx^b$, where $a <$ b are positive integers and u, v belong to the set $\{-1, 1\}$ (so, in particular, every Newman trinomial $1+x^a+x^b$) divides some Littlewood polynomial. Nevertheless, we prove that there exist Newman polynomials which divide no Littlewood polynomial, e.g., $x^9 + x^6 + x^2 + x + 1$. This example settles the problem 006:07 posed by first named author at the 2006 West Coast Number Theory conference. It also shows that the sets of roots of Newman polynomials V_N, Littlewood polynomials V_L and $\{-1, 0, 1\}$-polynomials V are distinct in the sense that V_N is not a subset of V_L, and V is not equal to the union of the sets V_L and V_N.

Fractions and approximation in a non-integer base.

We explain how to construct a number system in a non-integral base. Then we try to adapt several well-known results of diophantine approximation on this framework when the chosen base satisfies suitable algebraic properties

On the difference between two Mahler measures.

We prove that every real algebraic integer α can be written as $\alpha = M(P) - M(Q)$, where $P(X)$ and $Q(X)$ are integer polynomials of the same degree as α, and M denotes the Mahler measure.