ERC Advanced Grant 2022

Research proposal [Part B]

Integrating Spectral and Geometric data on Moduli Space

InSpeGMos

Cover Page.

- **Principal Investigator:** Nalini Anantharaman
- **Host Institution:** University of Strasbourg
- **Proposal duration:** 60 months

Each physical object possesses specific frequencies of vibrations, called its "eigenfrequencies", at which it enters in resonance under an external stimulus. In mathematical terms these frequencies are the "eigenvalues" of a linear operator; they form the "spectrum" of the object. Spectral geometry is concerned with understanding how the spectrum of an object, as well as the modes of vibration (eigenfunctions) associated to each eigenfrequency, are related to its geometric shape. This is a wide area of research, with applied and interdisciplinary aspects (electromagnetic waves, vibrating solids, seismic waves, wave functions in quantum mechanics...), but also involving very theoretical mathematics, with many natural questions still open: What can we learn about the topology or geometry of an object by observing its spectrum? Can we predict if the vibrations will be localized in a small part of the object or on the contrary, if they will take place everywhere ? Can we construct an object and be sure that certain frequencies are in the spectrum, or, on the opposite, be sure to avoid certain sets of frequencies ? Can there be objects of arbitrarily large size, with no small eigenfrequencies ? Project InSpeGMos deals with a specific mathematical model : hyperbolic surfaces. The *Moduli Space* is a space of parameters of these surfaces that we can tune, and observe how the geometry and the spectrum vary. In the semiclassical regime (when the wavelength is small compared to the size of the object), it is expected that certain spectral features are universal. We will adopt a probabilistic point of view: try to exhibit spectral and geometric phenomena that happen in 99% of cases. The project is focussed on developing new integration techniques on Moduli Space. We shall look for new coordinates, generalize Mirzakhani's study of volume functions, and seek inspiration in Random Graph Theory to develop new probabilistic methods in the spectral theory of Random Surfaces.

Section a. Extended synopsis of the scientific proposal

Genesis : 3 conjectures in Quantum Chaos. To this day, the main geometric tool to understand the propagation of waves remains the semiclassical approximation : it tells us that wave fronts move in straight lines (or in a curved space, geodesics). For instance, the propagation of light, which is an electromagnetic wave, is often treated by geometric optics, where the trajectory of light is described by light rays. It is seen in experiments that the semiclassical approximation is pertinent, in many respects, to explain the geometric patterns observed in waves. In particular, when the dynamics of the geodesic flow is chaotic, this classically chaotic behaviour seems to persist in wave propagation : this is called "Quantum Chaos". Unfortunately the ideas of Quantum Chaos are mostly expressed as conjectures, which remain to be proven mathematically. Today it seems necessary to find new patterns of thinking to make progress. A typical geometric situation where the following conjectures should apply, are *hyperbolic manifolds* and the spectrum of the laplacian, the main object of study of project InSpeGMos.

Random Matrix Conjecture (Bohigas-Giannoni-Schmit) [12]: the spectrum of vibration resembles statistically the eigenvalues of a large random gaussian symmetric matrix (Wigner statistics). There is absolutely no mathematical progress to prove this conjecture; but one paper in the physics literature (Sieber-Richter [47]) gives a hint for an analytical approach on hyperbolic surfaces. The analytic tools that this project InSpeGMos aims at developing, should be useful to give ultimately a rigourous mathematical interpretation to the approach of [47, 46].

Quantum ergodicity conjecture [45]. This conjecture is concerned with the eigenfunctions, more precisely, the probability density defined by their square. This probability should, conjecturally, approach in the semiclassical limit the uniform probability on phase space (also called microcanonical measure). The PI's former work has been largely focussed on the Quantum Ergodicity phenomenon, however this theme is marginal in the current application.

Berry conjecture [11]. This conjecture is again concerned with the eigenfunctions. It appears that they should "look like" a monochromatic Gaussian random process. A more precise (but debated) formulation is first to apply a magnifying zoom to make the wavelength 1, and to focus on a randomly chosen ball [24]. This interpretation stems from a result of Backhausz-Szegedy [6], which I interprete as establishing the Berry conjecture on random graphs.

The paper [6], together with [2, 29], play an important role in the genesis of this project, as they suggest **possible cross-fertilization of ideas between the domains of spectral theory of graphs (random or not) and spectral geometry on manifolds (with a need to define what is a random manifold)**.

Random geometries. The progress regarding the 3 conjectures above is stalled : one must try new point of views, even if it means departing from the original setting. It is an appealing idea to study these conjectures for a randomly chosen dynamical system : maybe we can prove results that are valid in 99 % of cases. It could, in fact, help avoid certain pathological situations that are difficult to name explicitly. The issue is not to define models of random systems, but to find ones where computations are actually possible. Here we will focus on models of random hyperbolic surfaces. The goal of this project is not to solve any of the 3 conjectures within 5 years, but to develop new tools of calculations that will be useful for a whole community of mathematicians who want to explore these questions.

We also want to winkle out the semiclassical approximation from its traditional setting (the regime of small wavelengths), and take it to a new setting where we study the spectrum of families of large geometric objects. This has been suggested before in the physics literature concerning quantum chaos on graphs [27, 28, 48, 49]. In mathematics the idea to study Quantum Ergodicity of eigenfunctions in the large volume limit has been initiated by Anantharaman-LeMasson [2] for large graphs and LeMasson-Sahlsten for large hyperbolic surfaces (equivalently, surfaces of large genus g) [29]. See also Anantharaman-Sabri-Ingremeau-Winn [5, 4, 1] for recent developments.

A couple of models of random hyperbolic surfaces have been introduced. In this project we primarily focus on the Weil-Petersson probability measure on Moduli Space, for which we can rely on Mirzakhani's fundamental work [37]. This is a continuous probabilistic model, where we can tune certain lengths and angles to vary the geometry of the surface. We will also, when relevant, study other models such as the Brooks-Makover model [15] or the random cover models [33, 32, 34]. Our project is made timely by recent advances such as [37] or [40]; but in spite of those, an obstacle towards the ambitious goals proposed above, is the lack of techniques to compute expectations and probability of geometric events. This project mainly aims at developing such techniques : both for surfaces of fixed topology, and in the large genus asymptotics $g \longrightarrow +\infty$.

Beyond Quantum Chaos, here is a sample of questions of fundamental interest about random surfaces; they are of two natures, geometric and spectral. The Selberg trace formula naturally relates the two aspects.

(1) Spectral statistics, in particular spacings between eigenvalues. Given an interval $I \subset \mathbb{R}$, can we obtain information on the k-point correlation function of eigenvalues of the Laplacian lying in I ? In particular, do they obey the statistics of the Wigner ensemble ?

(2) Low eigenvalues. It is a central goal of spectral geometry to be able to provide information about the absence or presence of spectrum in a given region. For hyperbolic surfaces, the interval $(0, 1/4)$ at the bottom of the spectrum plays a special rôle in many respects. By work of Mirzakhani [39], there is $c > 0$ such that, with probability going to 1, the interval $(0, c)$ is free of eigenvalues. Recent results show that one can take $c = 3/16 - \epsilon$ for any $\epsilon > 0$ ([30, 52] for the Weil-Petersson model, [34] for the random cover model). Can we push this to $c = 1/4 - \epsilon$? The value $1/4 - \epsilon$ has been obtained for random-covers of a non-compact surface of finite volume [32].

(3) Cheeger constant. The Cheeger constant $h(X)$ of a Riemannian manifold X is an effective measure of the connectedness of a manifold. In [39], it shown that with probability going to 1 as $g \longrightarrow +\infty$, $h(X) > \frac{\log 2}{2\pi + \log 2}$, but the optimal constant is unknown.

(4) L^∞ norms of eigenfunctions, delocalization, Quantum Unique Ergodicity. "Delocalization" of a wave function φ roughly means that the probability density $|\varphi(x)|^2$ does not concentrate in a small part of the manifold. Ideally, the goal is to show that $|\varphi_k(x)|^2$ is "close" to being constant if φ_k is an eigenfunction, in the semiclassical regime $\lambda_k \longrightarrow +\infty$. A first approach in this direction is to bound from above the L^∞ norm. For a compact Riemannian surface X (without any geometric assumption), it is known that $\|\varphi_k\|_\infty = \mathcal{O}_X(\lambda_k^{1/2})$. Can we substantially improve this bound for a typical hyperbolic surface X ?

Another approach to delocalization is via Quantum (Unique) Ergodicity : a test function $a(x)$ is chosen and one tries to show that $\int_X a(x)|\varphi_k(x)|^2 dx$ is close to $\int_X a(x)dx$. For any hyperbolic surface, this has been proven for a majority of eigenfunctions, both for large frequency λ_k [50] and for large volume [29]. However, for a random surface one expects much more : instead of having a statement for "almost all" eigenfunctions, one would expect a result for all eigenfunctions, similar to recent advances on random graphs and random matrices models [14, 20, 21, 19, 10, 8, 9, 23].

(5) Topology of long closed geodesics for high-genus surfaces. Numerous results have been obtained concerning the relation between length and topology (e.g. number of self-intersections) of closed geodesics, in deterministic as well as probabilistic contexts [16, 38, 36, 26, 25, 7, 40, 42]. Having in mind the Selberg trace formula which converts the length spectrum to the spectrum of the laplacian, one needs to examine the transitions between the different regimes when the length L, the number of self-intersections k and the genus g all grow to $+\infty$ in an inter-related way.

(6) Bers constant. Every hyperbolic surface has a decomposition into pairs of pants, whose boundary geodesics all have lengths $\leq L_g \leq 26(g-1)$ (see [16]). A quantity L_g having this property is called a "Bers constant", but the optimal value is not known. Any probabilistic improvement of the known deterministic linear upper bound $L_g \leq 26(g-1)$ would be essential in understanding the spectral geometry of random surfaces.

(7) Gaussian value distribution for eigenfunctions For a typical hyperbolic surface and an eigenfunction φ_k, can we show that the value distribution of φ_k is close to a Gaussian ? The question can be asked in different regimes : fixed g and $\lambda_k \longrightarrow +\infty$ (as in the Berry conjecture); bounded λ_k and $g \longrightarrow +\infty$; or both $g, \lambda_k \longrightarrow +\infty$.

Some of these questions have been recently solved for random regular graphs [10, 8, 9, 23, 22, 13, 23, 6]. Results for graphs always rely on the fact that we have discrete probability spaces, with sophisticated techniques to estimate the cardinalities and probabilities, and concentration inequalities on large product spaces. For the Weil-Petersson model, we must admit that the techniques of integration and estimation of probabilities are exceedingly few. One of the main reasons is that the Moduli Space is defined as a quotient space : the quotient of Teichmüller space by the Mapping Class Group. This difficulty is partially overcome by formulas of McShane, Mirzakhani, Luo-Tan [35, 37, 31], which yield for instance explicit recursive formulas for the total volume of Moduli Space [37]. These results, as well as those of Wolpert [51], according to which the Fenchel-Nielsen coordinates are canonical coordinates, are the two main techniques of integration. The existing techniques do not allow to compute expectations of many basic indicator functions. : just to name one example, we do not have a satisfactory estimate of the probability for the surface to have a decomposition into pairs of pants of boundary lengths $\leq L$. **This is why we are convinced of the necessity to diversify the existing techniques to integrate "geometrically defined" random variables on Moduli Space**. For this project we have identified the following specific goals.

Goal 1. Develop a theory of volume functions. Let S be a fixed compact orientable surface; fix a closed curve γ. If X is a hyperbolic structure on S, we denote by $L_X(\gamma)$ the length of the unique closed geodesic in the homotopy class of γ, when S has the hyperbolic structure X. One of the most basic functions on Teichmüller space is the function $L_\gamma : X \mapsto L_X(\gamma)$ (as well as the periodization of this function by the Mapping Class Group, which defines a function on Moduli Space). Let us call μ_γ the pushforward of the Weil-Petersson measure under L_γ : it has a density $\mu_\gamma(d\ell) = V_\gamma(\ell)d\ell$. Being able to integrate functions of L_γ is equivalent to knowing the density V_γ explicitly. Mirzakhani showed how to determine this density if γ is a multi-curve, that is, a non-intersecting union of simple curves. Our first goal is to describe the density V_γ, when γ is a curve with self-intersections (or a union of curves that may intersect). More generally, instead of periodic geodesics, we would like to formulate and study similar questions concerning closed piecewise geodesics. Not being able to deal with piecewise geodesics is a major obstacle to the adaptation of certain results about random graphs.

Goal 2. Study the bottom of the spectrum of random hyperbolic surfaces. A second objective of the project is to study the low eigenvalues of the laplacian on a random hyperbolic manifold. Our main focus will be on the limit $g \longrightarrow +\infty$, for which we would like to prove that for any $\epsilon > 0$, there are no eigenvalues in the interval $(0, 1/4 - \epsilon)$, with probability going to 1. During L. Monk's PhD thesis, we envisioned a plan of attack and can now divide the progress towards this goal into several tasks explained later.

Goal 1 and 2 are the first goals of this project, reachable within 5 years. The next goals 3, 4 are more distant goals, sought for by a whole community of mathematicians and mathematical physicists, we mention them here as part of the general landscape. They will be a source of inspiration, and we believe that the integration techniques we will develop will help make partial progress, profitable to several communities working on random geometries, Teichmüller theory, spectral theory and quantum chaos. Goal 5 is a more concrete extraction from Goal 4.

Goal 3. Understand the spectral statistics of random hyperbolic surfaces in the "bulk" of the spectrum. L. Monk [42, 43], in her thesis supervised by the PI, proved that the density of eigenvalues of a random surface of genus g converges (as g grows to infinity) to the spectral density of the hyperbolic plane, using the notion of Benjamini-Schramm convergence. The number of eigenvalues in an fixed interval is typically of order g. Monk could give effective rates of convergence, using quantitative knowledge of the geometry of random surfaces, and go to scales slightly smaller than the unit scale. The natural next step is to try to diminish this scale as close as possible to the mean spacing, ideally by counting the eigenvalues in intervals of size $1/g^{1-\epsilon}$. The other direction to explore is to study spectral 2-point correlations, which appear as the square of the error estimated in [43], or the square of the geometric term of the Selberg trace formula. It is expected that the behaviour is universal and corresponds to that of Wigner's Gaussian Orthogonal Ensemble (GOE). Many heuristic arguments in mathematics and physics, always using the square of some trace formula, have been suggested [47, 46], but have so far failed to give a rigorous proof. One reason is that the heuristics were claimed to hold for a "generic" surface, without a precise mathematical formulation. The Weil-Petersson probabilistic setting can be a solution to this issue, by allowing to average over the set of surfaces. We can use the powerful tool-set developed by Mirzakhani, but will also need to enrich it with new techniques : for instance, we need to evaluate moments of random variables involving lengths of non-simple geodesics, cf. Goal 1.

Goal 4 : prospective. Explore techniques available for random graphs models, not available for Weil-Petersson model. Today the main tool used to study the spectral statistics of hyperbolic surfaces remains the Selberg trace formula, giving a relation between the spectrum of the laplacian and the lengths of periodic geodesics. One always starts by estimating the probability of geometric events (involving lengths of periodic geodesics) and then translates into spectral results. Because of the uncertainly principle, going to smaller spectral intervals means studying longer periodic geodesics, and having to deal with their exponential proliferation. This approach is limited by the mere fact that Mirzakhani's techniques are limited to simple geodesics or multicurves. Longer geodesics will have more and more self-intersections, and completely escape the domain of application of Mirzakhani's formulas.

In spectral graph theory and random matrix theory, there are many other tools that don't exist yet for random surfaces, in fact it is not even clear what the adaptation of these techniques to continuous geometries could be. To name a few of these :

– the fixed point equation satisfied by the semi-circle law; the Schur complement formula for matrices;

– the use of "geometric" measure preserving transformations, such as "switching" of edges, to obtain fixed point equations or concentration of measure phenomena [6, 9, 10];

– the Dyson Brownian motion to show convergence of spectral statistics to GOE [17, 18]

– counting techniques, use of entropy to quantify the probability of typical or atypical events (large deviations) [6]

Currently there are few ideas to implement these ideas in the case of random manifolds, but we will be in constant prospective to do so. The following goals seems achievable :

Goal 5. Study of the Moduli Space as a large-dimensional probability space. Concentration of measure phenomena. Large deviations. On product spaces of growing dimensions, concentration of measure phenomena tell us that random variables that are not very sensitive to the variation of a few coordinates, are sharply concentrated around their means. The Moduli Space of compact hyperbolic surfaces of genus g is not a product space, but can be identified with one by choosing local coordinates. We will seek to formulate concentration of measure phenomena. We also hope to develop a theory of "typicality" for functions defined on a random hyperbolic surfaces of large genus, similar to what was done in [6] : use a notion of entropy to describe the probability distribution of "typical" functions (after defining them).

The project is at the same time *high risk* and *high gain*, for a common reason : few of these questions have been explored before, in particular, the distribution of lengths of non-simple geodesics, which is the core of the project. Each of the goals is ambitious, but the work can be divided into intermediate steps which are sure to bring novel information. We divide the progress towards Goals 1, 2, 5 into the following list of concrete tasks to be achieved within 5 years. Meanwhile we will constantly keep in mind potential advances towards Goals 3, 4 or any of the open questions listed above.

Task 1. Explore new coordinates systems on Teichmüller space. In order to find new ways of integrating over Moduli Space, we first need to find new ways of integrating on Teichmüller space itself. So far, one can only integrate functions that admit a nice expression in at least one system of Fenchel-Nielsen coordinates. We will seek to find new nice coordinate systems on Teichmüller space (involving lengths of curves, distances, angles,...).

Task 2. Study of generalized volume functions. Theory of generalized convolution. Our Goal 1 is to study the generalized volume functions V_γ, when γ is a curve with self-intersections. Since the curve γ with self-intersections may be seen as a geometric "concatenations" of simple closed curves $\gamma_1, \ldots, \gamma_k$ (in a sense to be made precise), and the length of this " concatenation" of curves is close to being the *sum* of the lengths of the original curves, the functions V_γ should be obtained from the V_{γ_j} a certain type of algebraic operation resembling a convolution. We will develop a theory of generalized convolutions : pushforward of Lebesgue measure under operations resembling a sum.

The densities V_{γ_j} corresponding to simple curves are polynomials [37]. Polynomials are stable under convolutions, so we would expect V_γ to resemble a polynomial. We need to determine a class of functions, containing polynomials, and stable under the operation of generalized convolution that we will have defined. This will be done for fixed g in the limit $\ell \longrightarrow +\infty$, and we have in mind that the functions V_γ could belong to one of Hörmander's classical symbol spaces (functions that have an expansion in powers of ℓ, as well as their derivatives). To develop this technique, having good coordinates on Moduli Space will be absolutely necessary, so developing Task 2 should be a follow-up of Task 1.

Task 3. Asymptotics of volume functions in large genus. Ramanujan functions. We want to study the structure of the volume functions $V_\gamma(\ell)$ for γ a curve of fixed topological type. After having described them as general convolutions of polynomial functions of ℓ, and in parallel to studying their behaviour for fixed g in the limit $\ell \longrightarrow +\infty$ (Task 2), the next task is to study their behaviour in the limit $g \longrightarrow +\infty$. A first goal would be to prove the existence of an asymptotic expansion in inverse powers of g (like the one obtained in [41, 3]) : **(1)** $V_\gamma(\ell) \sim \sum_{j=0}^{+\infty} g^{-j} V_\gamma^{(j)}(\ell)$, valid to any given order. Thereafter our main goal will be to understand the structure of the coefficients $V_\gamma^{(j)}(\ell)$ in this expansion. We will seek inspiration in the work of J. Friedman to prove the Alon conjecture about the optimal spectral gap for random regular graphs [22]. In the context of regular graphs he introduced "Ramanujan functions" and proved that the $V_\gamma^{(j)}(\ell)$ belong to this class of functions. This is one of the main technical innovations we want to develop to achieve Goal 2.

Task 3'. Explore the possibility to extend the questions and methods of Task 3 to other random models, such as the Random Cover model [40].

Task 4. Describe the average of the trace formula over Moduli Space : sum over infinitely many topological types of closed geodesics. Once we have proven **(1)** for any given topology of γ, we need to sum over all γ, if we want to study the average of the geometric term in the Selberg Trace Formula and deduce results about the spectrum (e.g. Goals 2, 3). While it seems plausible that an asymptotic expansion like **(1)** still exists after summing over all γs, it is unlikely that the sum $\sum_\gamma V_\gamma^{(j)}(\ell)$ still has the Ramanujan property. The lesson taught in [22] or [13] is that it is necessary to discard a set of bad surfaces, before averaging on Moduli Space. The work of Monk-Thomas [44] should be useful to define precisely those bad surfaces (the ones which contain geodesic "tangles").

Task 5. Extend further our study of generalized volume functions (to include n-tuples of closed curves, or broken geodesics). In order to study higher order correlations of laplacian eigenvalues, a common idea is to take powers of both sides of the Selberg trace formula. For instance, if we take the square of the trace formula, we obtain a relation between pairs of eigenvalues and pairs of (lengths of) periodic geodesics. It is thus relevant to extend all the questions above to find a description, as accurate as possible, of the densities $V_\gamma(\ell)$ when γ is an n-tuple of periodic geodesics. We would also like to explore similar questions when γ is a broken geodesic (piecewise geodesic path). This would be essential, for instance, to obtain probabilistic local Weyl laws, using the pre-trace formula

Task 6. Formulate and prove concentration of measure results on Moduli Space. Describe the probability of rare events via entropy / large deviations. The difficulty towards Goal 5 is that the Moduli Space is not a product space : we need to work in local coordinates. We will use random local coordinates, for instance by choosing at random a decomposition into 'pairs of pants" and the associated Fenchel-Nielsen coordinates.

Section b. Curriculum vitae

Personal Information.

Name:	Anantharaman, Nalini
ORCID:	0000-0002-8496-9382
Date of birth:	February 26, 1976
Nationality:	French
URL for web site:	https://irma.math.unistra.fr/~anantharaman

Degrees.

2006	Habilitation (defended Dec 2006) Université Claude-Bernard / E.N.S. Lyon, France
2000	PhD in Mathematics (defendedSep 2000) Université Pierre-et-Marie-Curie, Paris, France Thesis advisor: François Ledrappier (Laboratoire de Probabilités)

Positions.

2022–present	Professor, Collège de France (Paris), Member of USIAS (University of Strasbourg Institute for Advanced Study).
2014–2022	Professor, Institut de Recherche Mathématique Avancée (IRMA), Université de Strasbourg, France Member of USIAS (University of Strasbourg Institute for Advanced Study).
2009–2014	Professor, Université Paris-Sud, Orsay, France.
2006–2009	CNRS Researcher (CR1), Associate "Hadamard" Professor Centre de Mathématiques Laurent Schwartz, École Polytechnique, Palaiseau
2001–2006	Maître de Conférences, UMPA, Ecole Normale Supérieure, Lyon, France

Honors and awards.

2020	Frederic Esser Nemmers Prize in Mathematics
2019	Infosys prize in Mathematics
2019	Elected to the French Académie des Sciences
2013	Silver medal of the Centre National de la Recherche Scientifique (CNRS)
2012	Henri Poincaré prize
2011	Grand prix Jacques Herbrand de l'Académie des Sciences
2010	Salem prize
2007	Prix Gabrielle Sand et Marie Guido Triossi de l'Académie des Sciences

Selection of invited lectures, named fellowships.

2018	ICM plenary speaker (Rio de Janeiro)
2018	Zygmund-Calderón lectures (University of Chicago)
2017	Nachdiplomvorlesungen (ETH Zürich)
2016	Aisenstadt lectures (CRM Montéal)
2015	Bellow lectures (Northwestern University)
2015	ICMP plenary speaker (Santiago de Chile)
2010	ICM invited speaker
2008	ECM invited speaker
2006	ICMP invited speaker (Rio de Janeiro)
Fall 2019	Eisenbud professor, MSRI, Berkeley, USA.
Spring 2015	Eisenbud professor, MSRI, Berkeley, USA.
Spring 2013	Fellow of the Institute of Advanced Study, Institute of Advanced Study, Princeton USA.
Spring 2009	Miller Visiting Professor, University of California, Berkeley, USA.

Fellowships, grants.

2021–2028	Scientific coordinator of the project *Institut Thématique Interdisciplinaire IRMIA++*, Collective grant attributed by the University of Strasbourg (140 people), 6,2 MEuro.
2021–2025	Co-I of the research group (16 people) *Aléatoire, Dynamique et Spectre*, Grant ANR-20-CE40-0017 coordinated by G. Rivière (Univ. de Nantes), 255 744 euro.
2013–2017	Co-I of the research group (10 people) *Spectral Geometry, Graphs and Semiclassics*, Grant ANR-13-BS01-0007 coordinated by L. Hillairet (Univ. d'Orléans), 231 920 euro.
2009–2013	Co-I of the research grand *Méthodes spectrales en chaos classique et quantique* ANR-09-JCJC-0099-01 coordinated C. Guillarmou (E.N.S. Paris)
2005–2008	Co-I of the research group (7 people) *Résonances et décohérence en Chaos Quantique*, Grant ANR-05-JCJC-0107-01 coordinated by S. Nonnenmacher (CEA Saclay)
2012–2019	Member of Institut Universitaire de France

Institutional or editorial responsibilities.

Editorial board of Annales Sci. ENS (2010-2016), Journal of the Ramanujan Mathematical Society (2013-2018), Mathematische Annalen (2014-2016), Annales de la Faculté des Sciences de Toulouse (as of 2015), Annales Henri Lebesgue (as of 2017), Proceedings of the London Mathematical Society (2019–2021), Probability and Mathematical Physics (as of 2019).

Vice-president of the Société Mathématique de France (2010–2012). Member of Conseil National des Université (CNU), mathematics section (2015–2019).

Member of the scientific board of Institut Henri Poincaré (IHP, 2013-2019), Mathematische Forschung Oberwolfach (MFO, as of 2017), cluster of Excellence Mathematics Münster (as of 2019).

Director of the LabEx (Laboratoire d'Excellence) IRMIA, Strasbourg (as of February 2018).

Member of the French Académie des Sciences.

Member of the ICM Structure Committee.

Supervision of graduate students and postdoctoral fellows.

Supervisor of 8 PhD theses at École polytechnique, Université Paris-Sud, ENS Paris or Université de Strasbourg : G. Rivière, *Délocalisation des mesures semiclassiques pour des systèmes dynamiques chaotiques* (2009), É. Le Masson, *Ergodicité et fonctions propres du laplacien sur les grands graphes réguliers* (2013), Y. Bonthonneau, *Les résonances du Laplacien sur les variétés à pointes*, co-supervised with C. Guillarmou (2015), G. Klein, *Stabilisation et Asymptotique spectrale de l'équation des ondes amorties vectorielle* (2018), A. Deleporte, *The low-energy spectrum of Toeplitz operators* (2019), L. Monk, *Le bas du spectre des surfaces hyperboliques aléatoires*, (2021), D. Sanchez, *The bottom of the spectrum of random Schrödinger operators*, (to be defended in 2022).

Supervisor of 3 post-doctoral fellowships, at Université Paris-Sud or Université de Strasbourg : M. Ingremeau, *Quantum Ergodicity on Quantum graphs*, Labex IRMIA (2017-2018), M. Sabri, *Quantum Ergodicity for Schrödinger operators on large graphs*,Labex IRMIA (2015-2017), M. Léautaud, *Control and stabilization for the Schrödinger equation on the torus*, Paris-Sud Orsay (2011-2012).

References

[1] Anantharaman (Nalini), Ingremeau (Maxime), Sabri (Mostafa) & Winn (Brian) – *Quantum ergodicity for expanding quantum graphs in the regime of spectral delocalization*, J. Math. Pures Appl. (9), t. **151** (2021), pp. 28–98.

[2] Anantharaman (Nalini) & Le Masson (Étienne) – *Quantum ergodicity on large regular graphs*, Duke Math. J., t. **164** (2015), no. 4, pp. 723–765.

[3] Anantharaman (Nalini) & Monk (Laura) – *A high-genus asymptotic expansion of weil-petersson volume polynomials*, J. Math. Phys. (In press).

[4] Anantharaman (Nalini) & Sabri (Mostafa) – *Recent results of quantum ergodicity on graphs and further investigation*, preprint, arXiv:1711.07666 (2017).

[5] — , *Quantum ergodicity on graphs: from spectral to spatial delocalization*, Ann. of Math. (2), t. **189** (2019), no. 3, pp. 753–835.

[6] Backhausz (Ágnes) & Szegedy (Balázs) – *On the almost eigenvectors of random regular graphs*, Ann. Probab., t. **47** (2019), no. 3, pp. 1677–1725.

[7] Basmajian (Ara) – *Universal length bounds for non-simple closed geodesics on hyperbolic surfaces*, J. Topol., t. **6** (2013), no. 2, pp. 513–524.

[8] Bauerschmidt (Roland), Huang (Jiaoyang), Knowles (Antti) & Yau (Horng-Tzer) – *Bulk eigenvalue statistics for random regular graphs*, Ann. Probab., t. **45** (2017), no. 6A, pp. 3626–3663.

[9] Bauerschmidt (Roland), Huang (Jiaoyang) & Yau (Horng-Tzer) – *Local Kesten-McKay law for random regular graphs*, Comm. Math. Phys., t. **369** (2019), no. 2, pp. 523–636.

[10] Bauerschmidt (Roland), Knowles (Antti) & Yau (Horng-Tzer) – *Local semicircle law for random regular graphs*, Comm. Pure Appl. Math., t. **70** (2017), no. 10, pp. 1898–1960.

[11] Berry (Michael) – *Regular and irregular semiclassical wavefunctions*, J. Phys. A, t. **10** (1977), no. 12, pp. 2083–2091.

[12] Bohigas (Oriol) – *Random matrix theories and chaotic dynamics*, in *Chaos et physique quantique (Les Houches, 1989)*, North-Holland, Amsterdam, 1991, pp. 87–199.

[13] Bordenave (Charles) – *A new proof of Friedman's second eigenvalue Theorem and its extension to random lifts*, preprint (2015).

[14] Bourgade (P.) & Yau (H.-T.) – *The eigenvector moment flow and local quantum unique ergodicity*, Comm. Math. Phys., t. **350** (2017), no. 1, pp. 231–278.

[15] Brooks (Robert) & Makover (Eran) – *Random construction of Riemann surfaces*, J. Differential Geom., t. **68** (2004), no. 1, pp. 121–157.

[16] Buser (Peter) – *Geometry and spectra of compact Riemann surfaces*, Progress in Mathematics, vol. 106, Birkhäuser Boston, Inc., Boston, MA, 1992.

[17] Erdős (László), Schlein (Benjamin) & Yau (Horng-Tzer) – *Universality of random matrices and local relaxation flow*, Invent. Math., t. **185** (2011), no. 1, pp. 75–119.

[18] Erdős (László), Schlein (Benjamin), Yau (Horng-Tzer) & Yin (Jun) – *The local relaxation flow approach to universality of the local statistics for random matrices*, Ann. Inst. Henri Poincaré Probab. Stat., t. **48** (2012), no. 1, pp. 1–46.

[19] Erdős (László), Knowles (Antti), Yau (Horng-Tzer) & Yin (Jun) – *Spectral statistics of Erdös-Rényi graphs I: Local semicircle law*, Ann. Probab., t. **41** (2013), no. 3B, pp. 2279–2375.

[20] Erdős (László), Schlein (Benjamin) & Yau (Horng-Tzer) – *Local semicircle law and complete delocalization for Wigner random matrices*, Comm. Math. Phys., t. **287** (2009), no. 2, pp. 641–655.

[21] — , *Semicircle law on short scales and delocalization of eigenvectors for Wigner random matrices*, Ann. Probab., t. **37** (2009), no. 3, pp. 815–852.

[22] Friedman (Joel) – *A proof of Alon's second eigenvalue conjecture and related problems*, Mem. Amer. Math. Soc., t. **195** (2008), no. 910, pp. viii+100.

[23] Huang (Jiaoyang) & Yau (Horng-Tzer) – *Spectrum of random d-regular graphs up to the edge.*

[24] Ingremeau (Maxime) – *Local weak limits of Laplace eigenfunctions*, preprint (2017).

[25] Kifer (Yuri) – *Large deviations in dynamical systems and stochastic processes*, Trans. Amer. Math. Soc., t. **321** (1990), no. 2, pp. 505–524.

[26] — , *Large deviations, averaging and periodic orbits of dynamical systems*, Comm. Math. Phys., t. **162** (1994), no. 1, pp. 33–46.

[27] Kottos (Tsampikos) & Smilansky (Uzy) – *Quantum chaos on graphs*, Phys. Rev. Lett., t. **79** (1997), pp. 4794–7.

[28] — , *Periodic orbit theory and spectral statistics for quantum graphs*, Ann. Physics, t. **274** (1999), no. 1, pp. 76–124.

[29] Le Masson (Etienne) & Sahlsten (Tuomas) – *Quantum ergodicity and Benjamini-Schramm convergence of hyperbolic surfaces*, Duke Math. J., t. **166** (2017), no. 18, pp. 3425–3460.

[30] Lipnowski (Michael) & Wright (Alex) – *Towards optimal spectral gaps in large genus.*

[31] Luo (Feng) & Tan (Ser Peow) – *A dilogarithm identity on moduli spaces of curves*, J. Differential Geom., t. **97** (2014), no. 2, pp. 255–274.

[32] Magee (Michael) & Hide (Will) – *Near optimal spectral gaps for hyperbolic surfaces.*

[33] Magee (Michael) & Naud (Frédéric) – *Explicit spectral gaps for random covers of Riemann surfaces*, Publ. Math. Inst. Hautes Études Sci., t. **132** (2020), pp. 137–179.

[34] Magee (Michael), Naud (Frédéric) & Puder (Doron) – *A random cover of a compact hyperbolic surface has relative spectral gap* $\frac{3}{16} - \epsilon$.

[35] McShane (Greg) – *Simple geodesics and a series constant over Teichmuller space*, Invent. Math., t. **132** (1998), no. 3, pp. 607–632.

[36] Mirzakhani (Maryam) – *Counting mapping class group orbits on hyperbolic surfaces.*

[37] — , *Simple geodesics and Weil-Petersson volumes of moduli spaces of bordered Riemann surfaces*, Invent. Math., t. **167** (2007), no. 1, pp. 179–222.

[38] — , *Growth of the number of simple closed geodesics on hyperbolic surfaces*, Ann. of Math. (2), t. **168** (2008), no. 1, pp. 97–125.

[39] — , *Growth of Weil-Petersson volumes and random hyperbolic surfaces of large genus*, J. Differential Geom., t. **94** (2013), no. 2, pp. 267–300.

[40] Mirzakhani (Maryam) & Petri (Bram) – *Lengths of closed geodesics on random surfaces of large genus*, Comment. Math. Helv., t. **94** (2019), no. 4, pp. 869–889.

[41] Mirzakhani (Maryam) & Zograf (Peter) – *Towards large genus asymptotics of intersection numbers on moduli spaces of curves*, Geom. Funct. Anal., t. **25** (2015), no. 4, pp. 1258–1289.

[42] Monk (Laura) – *Geometry and spectrum of typical hyperbolic surfaces, phd thesis.*

[43] — , *Benjamini-schramm convergence and spectrum of random hyperbolic surfaces of high genus*, Analysis and PDE (In press).

[44] Monk (Laura) & Thomas (Joe) – *The tangle-free hypothesis on random hyperbolic surfaces*, IMRN (2021).

[45] Rudnick (Zeév) & Sarnak (Peter) – *The behaviour of eigenstates of arithmetic hyperbolic manifolds*, Comm. Math. Phys., t. **161** (1994), no. 1, pp. 195–213.

[46] Sieber (Martin) – *Semiclassical approach to spectral correlation functions*, in *Hyperbolic geometry and applications in quantum chaos and cosmology*, London Math. Soc. Lecture Note Ser., vol. 397, Cambridge Univ. Press, Cambridge, 2012, pp. 121–142.

[47] Sieber (Martin) & Richter (Klaus) – *Correlations between periodic orbits and their rôle in spectral statistics*, Physica Scripta, t. **T90** (2001).

[48] Smilansky (Uzy) – *Quantum chaos on discrete graphs*, J. Phys. A, t. **40** (2007), no. 27, pp. F621–F630.

[49] — , *Discrete graphs – a paradigm model for quantum chaos*, Séminaire Poincaré, t. **XIV** (2010), pp. 1–26.

[50] Šnirel′man (A. I.) – *Ergodic properties of eigenfunctions*, Uspehi Mat. Nauk, t. **29** (1974), no. 6(180), pp. 181–182.

[51] Wolpert (Scott) – *On the Weil-Petersson geometry of the moduli space of curves*, Amer. J. Math., t. **107** (1985), no. 4, pp. 969–997.

[52] Wu (Yunhui) & Xue (Yuhao) – *Random hyperbolic surfaces of large genus have first eigenvalues greater than* $\frac{3}{16} - \epsilon$.