Advanced Studies in Pure Mathematics 2?, 199?
Singularities and Arrangements, Sapporo-Tokyo 1998
pp. 1–15

Cobordism of non-spherical knots

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Abstract.

—We give a survey of the theory of cobordism for knots, first for algebraic knots and after for fibered non-simple knots. We also explain the construction of the first examples of cobordant but not isotopic non-spherical knots.—

§1. Introduction

In the 60's M. Kervaire [10] and J. Levine [12] have developed a theory of cobordism for spherical knots. We present a theory of cobordism for algebraic knots developed with F. Michel in [2], and more generaly for non-spherical knots. One motivation for this theory is the study of the topology of isolated singularities of complex hypersurfaces.

Let $f : \mathbf{C}^{n+1}, 0 \to \mathbf{C}, 0$ a holomorphic germ with an isolated singularity at the origin. The oriented preserving homeomorphism class of the pair $(D_\varepsilon^{2n+2}, f^{-1}(0) \cap D_\varepsilon^{2n+2})$ does not depend of the choice of ε small, it is the *topological type* of f. The oriented diffeomorphic class of the pair $(S_\varepsilon^{2n+1}, K_f)$ where $K_f = f^{-1}(0) \cap S_\varepsilon^{2n+1}$ is the *algebraic knot* associated to f. By Milnor's conic structure theorem [16] the algebraic knot associated to f determine the topological type of f, so we are interested in studying the topology of algebraic knots.

Recall that two algebraic knots K_0 and K_1 are *cobordant* (following M. Kervaire and J. Levine) if there exists a manifold K and an embedding $\Phi : K \times [0,1] \to S^{2n+1} \times [0,1]$ such that $\Phi(K \times \{0\}) = K_0$ and $\Phi(K \times \{1\}) = -K_1$ where $-K_1$ is the knot with the reversed orientation.

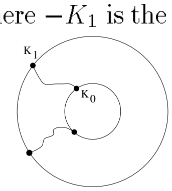

Fig. 1. Cobordism between K_0 and K_1.

Isotopy implies cobordism, moreover D.T. Lê [11] showed that for one dimensional algebraic knots cobordism implies isotopy. This is not true in higher dimensions since P. Du Bois and F. Michel [6] have constructed for all $n \geq 3$ some $(2n-1)$-dimensional algebraic spherical knots which are cobordant but not isotopic.

We can also study the cobordism in a general context. More precisely, a *knot* is a $(n-2)$-connected, oriented, smooth, closed, $(2n-1)$ dimensional submanifold of S^{2n+1}. A *spherical* knot is a knot abstractly homeomorphic to S^{2n-1}. For any knot K, there exists a smooth, compact, oriented $2n$-submanifold F of S^{2n+1}, having K as boundary ; such a manifold F is called a *Seifert surface* for K. For any $2n$ dimensional oriented smooth submanifold F of S^{2n+1}, we denote by G be the quotient of $\mathrm{H}_n(F)$[1] by its $\mathbf{Z}$-torsion. The *Seifert form* associated to F is the bilinear form $A : G \times G \to \mathbf{Z}$ defined as follows ; let (x, y) be in $G \times G$, then $A(x, y)$ is the linking number in S^{2n+1} of x and $i_+(y)$, where $i_+(y)$ is the cycle y "pushed" in $(S^{2n+1} \setminus F)$ by the positively oriented vector field normal to F in S^{2n+1}. By definition a *Seifert form for a knot* K is the Seifert form associated to a Seifert surface for K. A *simple knot* is a knot which has a $(n-1)$-connected Seifert surface. A knot K is a *simple fibered knot* if there exists a differentiable fibration $\varphi : S^{2n+1} \setminus K \to S^1$, being trivial on $U \setminus K$, where U is a "small" open tubular neighbourhood of K, and having $(n-1)$-connected fibers, the adherence of which are Seifert surfaces for K. Furthermore, Milnor's theory of singular complex hypersurfaces implies that algebraic knots are simple fibered knots

In §2 we define a new equivalence relation on the set of integral bilinear forms of finite rank called *algebraic cobordism*, in §3 we give a classification of simple fibered knots up to cobordism using algebraic cobordism of their Seifert forms, in §4 we explain how one can develop a theory of cobordism for fibered knots not necessary simple and in §5 we give the construction of cobordant but not isotopic non-spherical fibered knots.

§2. Algebraic cobordism

We define a new equivalence relation, called *algebraic cobordism* on the set $\mathcal{A}$ of bilinear forms defined on free $\mathbf{Z}$-modules G of finite rank. Let ε be $+1$ or -1. If A is in $\mathcal{A}$, let us denote by A^T the transpose of A, by S the ε-symmetric form $A + \varepsilon A^T$ associated to A, by $S^* : G \to G^*$ the adjoint of S (G^* being the dual $\mathrm{Hom}_{\mathbf{Z}}(G;\mathbf{Z})$ of G), by $\overline{S} : \overline{G} \times \overline{G} \to \mathbf{Z}$ the

[1] We denote by $\mathrm{H}_n(F)$ the nthhomology group of F with integer coefficients.

ε-symmetric non degenerated form induced by S on $\overline{G} = G_{/\mathrm{Ker}\, S^*}$. A submodule M of G is pure if $G_{/M}$ is torsion free. If M is any submodule of G let us denote by $M^\wedge$ the smallest pure submodule of G which contains M. In fact $M^\wedge$ is equal to $(M \otimes \mathbf{Q}) \cap G$. For a submodule M of G we denote by $\overline{M}$ the image of M in $\overline{G}$.

Definition 1. *Let $A : G \times G \to \mathbf{Z}$ be a bilinear form in $\mathcal{A}$. The form A is* Witt associated to 0 *if the rank m of G is even and if there exists a pure submodule M of rank $\frac{m}{2}$ in G such that A vanishes on M, such a module M is called a* metabolizer *for A.*

Definition 2. *Let $A_i : G_i \times G_i \to \mathbf{Z}$, $i=0,1$, be two bilinear forms in $\mathcal{A}$. Let G be $G_0 \oplus G_1$ and A be $(A_0 \oplus -A_1)$. The form A_0 is* algebraically cobordant *to A_1 if there exists a metabolizer M for A such that $\overline{M}$ is pure in $\overline{G}$, an isomorphism φ from $\mathrm{Ker}\, S_0^*$ to $\mathrm{Ker}\, S_1^*$ and an isomorphism θ from $\mathrm{Tors}\,(\mathrm{Coker}\, S_0^*)$ to $\mathrm{Tors}\,(\mathrm{Coker}\, S_1^*)$ which satisfy the two following conditions:*

(1) $M \cap \mathrm{Ker}\, S^* = \{(x, \varphi(x)); x \in \mathrm{Ker}\, S_0^*\}$, (c.1)

(2) $d(S^*(M)^\wedge) = \{(x, \theta(x)); x \in \mathrm{Tors}\,(\mathrm{Coker}\, S_0^*)\}$, (c.2)
where d is the quotient map from G^* to $\mathrm{Coker}\, S^*$.

Of course we have the following theorem:

Theorem 1. *[2, Theorem 1 p. 33]*
Algebraic cobordism is an equivalence relation on the set $\mathcal{A}$.

Remark. In the previous definition $S_i = A_i + \varepsilon A_i^T$ is the intersection form on $\mathrm{H}_n(F_i)$, $\mathrm{Ker}\, S_i^*$ is the image of $\mathrm{H}_n(K_i)$ in $\mathrm{H}_n(F_i)$ and $\mathrm{Coker}\, S_i^*$ is isomorphic to $\tilde{\mathrm{H}}_{n-1}(K_i)$. So for spherical knots, both $\mathrm{Ker}\, S_i^*$ and $\mathrm{Coker}\, S_i^*$ are zero, and conditions c.1 and c.2 vanish ; this corresponds to the classical situation of spherical knots studied by M. Kervaire and J. Levine.

§3. Cobordism of simple fibered knots

We have the following theorem :

Theorem 2. *[2, Theorem B p. 31]*
If $n \geq 3$, two algebraic knots, of dimension $2n-1$ are cobordant if and only if they have algebraically cobordant Seifert forms.

This theorem is a consequence of the two following theorems:

Theorem 3. *[2, Theorem 2′ p. 34]*
Let K_0 and K_1 be two simple fibered knots having F_0 and F_1 as $(n-1)$-connected fibers of differentiable fibrations. If K_0 is cobordant to K_1, then the Seifert forms A_0 and A_1, associated respectively to F_0 and F_1, are algebraically cobordant.

Proof. We have to construct a metabolizer which fulfills the conditions of definition 2. Let N be the compact, closed, oriented submanifold of $S^{2n+1}\times[0,1]$ obtained by gluing together (along their boundaries) F_0, the "tube" $\Phi(K\times[0,1])$ of the cobordism and F_1. By the classical obstruction theory it is easy to see that there exists a submanifold W of $S^{2n+1}\times[0,1]$ such that $\partial W = N$.

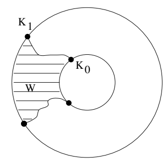

Fig. 2. The manifold W.

Let M be submodule of $\mathrm{H}_n(F_0)\oplus\mathrm{H}_n(F_1)$ which contains all the cycles of $\mathrm{H}_n(F_0)\oplus\mathrm{H}_n(F_1)$ which are boundaries in $\mathrm{H}_n(W)$. We are going to prove that $A = A_0 \oplus -A_1$ vanishes on M. Let $[a]$ and $[b]$ be two homology classes in M, thus there exists two $(n+1)$-chains x and y in W such that $\partial a = x$ and $\partial b = y$. Let i_+ be the positively oriented normal vector field to W in $S^{2n+1}\times[0,1]$. The intersection of x and $i_+(y)$ is zero, as shown in the following picture.

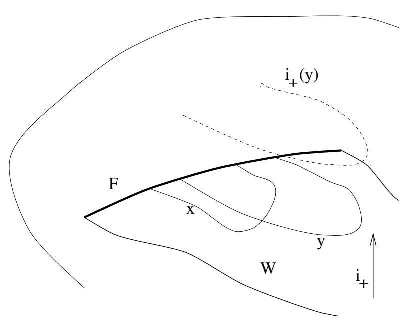

Fig. 3. Moving chains in W along i_+.

Hence the linking number in $S^{2n+1} \times \{0,1\}$ of a and $i_+(b)$ is zero. But this linking number is, by definition, equal to $A(a,b)$, so $A(a,b) = 0$ and $A_{|M} \equiv 0$.

To prove that M gives the algebraic cobordims, we must show that it fulfills the conditions of definition 2, this is quite hard and we refer to [2, §3] for details. Q.E.D.

Using classical methods of surgery, we will prove

Theorem 4. *[2, Theorem 3 p. 34]*
Let n be greater or equal to 3 and let K_0 and K_1 be two $2n-1$ dimensional simple knots. If the Seifert forms A_0 and A_1, associated to some $(n-1)$-connected Seifert surfaces F_0 and F_1 of K_0 and K_1, are algebraically cobordant then K_0 is cobordant to K_1.

Proof. First we do the connected sum, denoted by $\mathcal{S}$, of the two spheres in which K_0 and K_1 are imbedded, such that $K_0 \coprod K_1$ is a knot in this sphere $\mathcal{S}$. We also do the connected sum of the Seifert surfaces F_0 and F_1 in $\mathcal{S}$, such that this connected sum denoted by F is a Seifert surface for $K_0 \coprod K_1$. Let M be a metabolizer for $a = A_0 \oplus A_1$ as in definition 2. There exists (cf. [2, p. 36]) a basis $\mathcal{B} = \{m_i, m_i^*; {\scriptstyle i=1,\dots,s+r}\}$ of $\mathrm{H}_n(F_0) \oplus \mathrm{H}_n(F_1)$ such that:

(1) $\{m_i; {\scriptstyle i=1,\dots,s+r}\}$ is a basis of M,
(2) $\{m_i, m_i^*; {\scriptstyle i=s+1,\dots,s+r}\}$ is a basis of $\operatorname{Ker} S^*$ and $\{m_i^*; {\scriptstyle i=s+1,\dots,s+r}\}$ is a basis of $\operatorname{Ker} S_0^*$,
(3) the submodules $\langle m_i, m_i^* \rangle, {\scriptstyle i=1,\dots,s+r}$; are orthogonal for S, i.e. $\mathrm{H}_n(F_0) \oplus \mathrm{H}_n(F_1) = \bigoplus_{1 \le i \le s+r}^{\perp} \langle m_i, m_i^* \rangle$,
(4) when ${\scriptstyle i=1,\dots,s}$, $S(m_i, m_i^*) = a_i$.

Next we can do embedded surgeries in D^{2n+2} on all a basis of the metabolizer M at once, this gives a submanifold $\tilde{F}$ of D^{2n+2} with $\partial(\tilde{F}) = K_0 \coprod K_1$. On top of that we have $H_*(\tilde{F}, K_i) = 0$ for $i = 0, 1$, so according to the h-cobordism theorem (cf. [15]) $\tilde{F}$ gives the cobordism between K_0 and K_1. Q.E.D.

Remark. In Theorem 4 the definition of the algebraic cobordism of the Seifert forms gives a strategy to do surgery. Consider the case of a non spherical knot which is the disjoint union of two copies of S^{2n-1} with $S^{2n-1} \times [0,1]$ as a Seifert surface. This knot is cobordant to itself. If we do the connected sum of the two copies of $S^{2n-1} \times [0,1]$ then the metabolizer we obtain is of rank 2 with {a,b} as generators (see figure

4 below). There are two possible surgeries, as shown in the following picture one proves the cobordism the other does not.

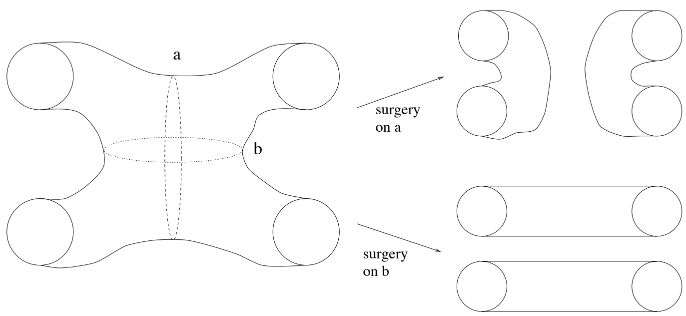

Fig. 4. Two surgeries are possible.

But the cycle b is in Ker $S^* \cap M$ and fullfils conditions of the algebraic cobordism between the two Seifert forms, and the cycle a does not.

§4. Cobordism of fibered knots

J. Levine (cf. [12, lemma 4 p.234]) proved: *Every* $(2n-1)$*-spherical knot is cobordant to a simple spherical knot.* We do the same in the case of non-spherical knots.

Proposition 1. *Every knot* K *is cobordant to a simple knot.*

Proof. First we choose a Seifert surface F for K, if F is not $(n-1)$-connected then we do embedded surgeries, in a $(2n+2)$-disk, on F to obtain a $(n-1)$-connected Seifert surface F' for K. (We refer to [4] for details.) Q.E.D.

Since we can realize an integral matrix as a Seifert form for a simple knot of dimension greater or equal to 5, we have the following:

Proposition 2. *Let* $n \geq 3$*. Let* K *be a* $(2n-1)$*-knot and* A *a Seifert form for* K*. Then* K *is cobordant to simple knot* K' *which has* A *as Seifert matrix.*

Proposition 1 allows us to prove the following theorems, which are the analogue of theorems 3 and 4.

Theorem 4. *Let $n \geq 3$. Let K_0 and K_1 be two $(2n-1)$-knots. if K_0 and K_1 have algebraically cobordant Seifert forms, then K_0 and K_1 are cobordant.*

Theorem 5. *Let $n \geq 3$. Let K_0 et K_1 be two $(2n-1)$-fibered knots, with A_0 and A_1 as Seifert forms. If K_0 and K_1 are cobordant then A_0 is algebraically cobordant to A_1.*

On top of that we have:

Theorem 6. *Let $n \geq 3$. Let K_0 and K_1 be two $(2n-1)$-fibered knots. The knots K_0 and K_1 are cobordant if and only if their Seifert forms are algebraically cobordant.*

Furthermore, having algebraically cobordant Seifert forms is also a necessary condition of cobordism for knots when n is 1 or 2. So we obtain, without any restriction of dimension, a "Fox-Milnor" relation (see [8]) for the Alexander polynomials of cobordant knots.

Let K be a $2n-1$ dimensional knot, and $\varepsilon = (-1)^n$. One can associate a polynomial $\Delta \in \mathbf{Z}[X]$ to any Seifert surface F for the knot K, defined by: $\Delta(X) = \det(XA + \varepsilon A^T)$, where A is the Seifert form associated to F. Such a polynomial Δ is called a Alexander polynomial for the knot K. Changing the Seifert surface to another multiplies Δ by $\pm X^m$ with m in $\mathbf{Z}$. For a polynomial γ in $\mathbf{Z}[X]$ we define the polynomial γ^* by: $\gamma^*(X) = X^{\deg \gamma}\, \gamma(X^{-1})$.

Proposition 3. *Let K_0 and K_1 be two cobordant $2n-1$ dimensional knots. If Δ_0 and Δ_1 are Alexander polynomials for K_0 and K_1, then there exists γ in $\mathbf{Z}[X]$ such that: $\gamma\,\gamma^* = \pm \Delta_0\, \Delta_1$.*

Proof. We denote by F_0 and F_1 two $(n-1)$-connected Seifert surfaces for K_0 and K_1, and by A_0 and A_1 the associated Seifert forms. The knots K_0 and K_1 are cobordant so proposition (3.10) implies that the form $A = A_0 \oplus -A_1$ has a metabolizer M. Therefore, there exists a basis for $\mathrm{H}_n(F_0) \oplus \mathrm{H}_n(F_1)$ such that in this basis the matrix for A is $\begin{pmatrix} 0 & B_1 \\ B_2 & B_3 \end{pmatrix}$ where B_i, i=1,2,3 are square matrices. We have $\Delta_0(X).\Delta_1(X) = \det(XA + \varepsilon A^T)$, hence $\Delta_0(X).\Delta_1(X) = \varepsilon.\det(XB_1 + \varepsilon B_2^T).\det(XB_2 + \varepsilon B_1^T)$. Let $\gamma(X)$ be $\det(XB_1 + \varepsilon B_2^T)$, then $\gamma^*(X) = \det(XB_2 + \varepsilon B_1^T)$. Finally we get $\gamma.\gamma^* = \pm\Delta_0.\Delta_1$. Q.E.D.

If F is the Milnor fiber of an algebraic knot K, then the associated Alexander polynomial is the characteristic polynomial of the monodromy. Hence the above proposition and the monodromy theorem imply the following proposition.

Proposition 4. *Let K_0 and K_1 be two algebraic knots having respectively Δ_0 and Δ_1 as characteristic polynomials of monodromy. If K_0 and K_1 are cobordant then the product $\Delta_0.\Delta_1$ is a square in $\mathbf{Z}[X]$.*

§5. Examples in the case of non-spherical knots

Proposition 5. *For all $n \geq 3$ there exits cobordant non-spherical fibered knots of dimension $2n-1$ which are not isotopic.*

Proof. Let us fix $n \geq 3$. We will use the spherical knots K_0 and K_1, of dimension $2n-1$ constructed by P. Du Bois and F. Michel in [6]. These knots are the first examples of cobordant and non isotopic algebraic spherical knots, now we will use them to construct some non spherical fibered knots which are cobordant but not isotopic.

Let K_i, with $i = 0, 1$, be the algebraic knot of dimension $2n-1$ associated to the isolated singularity at 0 of the germs of holomorphic functions $h_i : (\mathbf{C}^{n+1}, 0) \to (\mathbf{C}, 0)$ defined by:

$$h_i(x_0, \ldots, x_n) = g_i(x_0, x_1) + x_2^p + x_3^q + \sum_{k=4}^{n} x_k^2$$

with $g_0(x_0, x_1) = (x_0 - x_1)((x_1^2 - x_0^3)^2 - x_0^{s+6} - 4x_1x_0^{(s+9)/2})((x_0^2 - x_1^5)^2 - x_1^{r+10} - 4x_0x_1^{(r+15)/2})$, and $g_1(x_0, x_1) = (x_0 - x_1)((x_1^2 - x_0^3)^2 - x_0^{r+14} - 4x_1x_0^{(r+17)/2})((x_0^2 - x_1^5)^2 - x_1^{s+2} - 4x_0x_1^{(s+7)/2})$.

where $s \geq 11$ and $s \neq r+8$ are odd, and, p and q are distinct prime numbers which do not divide the product $\epsilon = 330(30+r)(22+s)$, [6, p.166]. We denote by A_i, $i = 0, 1$ the Seifert form associated to K_i defined on a free $\mathbf{Z}$-module of finite rank H_i.

Let L be the algebraic knot of dimension $2n-1$ associated to the isolated singularity at 0 defined by the germ:

$$\begin{array}{rccc} f: & (\mathbf{C}^{n+1}, 0) & \to & (\mathbf{C}, 0) \\ & (x_0, \ldots, x_n) & \mapsto & \displaystyle\sum_{k=0}^{n} x_k^2 \end{array}$$

according to [7, p.50] this algebraic knot has $A = ((-1)^{n(n+1)/2})$, defined on a free $\mathbf{Z}$-module of rank one G, as Seifert matrix.

We construct L_i the connected sum of L and K_i for $i = 0, 1$. The Seifert form for L_i is the integral bilinear form $A \oplus A_i$ defined on a free $\mathbf{Z}$-module $G_i = G \oplus H_i$ of finite rank. The knots L_i are simple fibered since $A \oplus A_i$ is unimodular and the knots L and K_i are simple. We denote by

S_i the $(-1)^n$-symetric form associated to $A \oplus A_i$, it is the intersection form for a fiber of L_i.We have $A = (\pm 1)$ so $\mathrm{Tors\,Coker} S_i^* \neq \{0\}$ or $\mathrm{Ker} S_i^* \neq \{0\}$; hence L_i, $i = 0, 1$ are not spherical knots.

Let M be the metabolizer for $A_0 \oplus -A_1$ given by P. Du Bois and F. Michel. The module $N = \Delta_G \oplus M$, where $\Delta_G = \{x \oplus x, x \in G\}$, is a metabolizer for $B = A \oplus A_0 \oplus -(A \oplus A_1)$. Since N fulfills c.1 and c.2 in definition 2 we have $A \oplus A_0$ algebraically cobordant to $A \oplus A_1$. So L_0 is cobordant to L_1 by theorem 4.

Now we are going to prove that the knots considered are not isotopic. Let τ_i be the monodromy associated to the fibered knot L_i, if there exists an integer e such that $(\tau_i^e - 1)\, G_i = 0$ then e is called an *exponent* for L_i. Recall that the *e-twist group* for L_i is defined as follows: assuming $(t^e - 1)^2\, G_i = 0$, if e is an exponent for L_i then the e-twist group associated to L_i is the group denoted by $GT^e(L_i)$ which is the $\mathbf{Z}$-torsion subgroup of the quotient $\mathrm{Ker}\,(t_i^e - 1)/_{(t_i^e - 1)H_i}$.

On top of that, we have $\epsilon = 330(30+r)(22+s)$ is an exponent for L_0 and L_1, and for all k which are multiple of ϵ the twist groups $GT^k(L_0)$ and $GT^k(L_1)$ have distinct orders. Finaly, as $\mathbf{Z}[t, t^{-1}]$-module $\mathrm{H}_n(G_0)$ and $\mathrm{H}_n(G_1)$ are not isomorphic. Hence the knots L_0 and L_1 are not isotopic. Q.E.D.

Remark. According to [1, th. 4 p. 117], the knots L_0 and L_1, which are the connected sum of two algebraic knots, cannot be algebraic.

§6. Questions

The methods used here are specific to dimensions greater than 5 (h-cobordism theorem, embedded surgery...), nevertheless since algebraic cobordism of Seifert forms is necessary in any dimension, we can ask:

Question 1. *What do we have to add to the definition of the algebraic cobordism of the Seifert forms in order to have the cobordism of 3-knots?*

Question 2. *Does cobordant but not isotopic 3-knots exist?*

Following a remark of O. Saeki, may be the definition of cobordism has to be changed. For instance we can use as a new definition of cobordism: two knots K_0 and K_1 are *weakly cobordant* if there exists a submanifold T of $S^{2n+1} \times [0, 1]$ such that $\partial(T) = K_0 \coprod -K_1$ where $-K_1$ is the knot with the reversed orientation and with $\mathrm{H}_*(T, K_0) = \mathrm{H}_*(T, K_1) = 0$.

Question 3. *Does algebraic cobordism of Seifert forms ossociated to 3-knots imply weakly cobordism of these 3-knots?*

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