

Contents

	<i>Preface</i>	<i>page</i> 1
	<i>Frequently Used Notation</i>	7
1	Distribution modulo one	10
	1.1 Weyl's Criterion	10
	1.2 Metrical results	14
	1.3 Discrepancy	18
	1.4 Distribution functions	20
	1.5 The multidimensional case	21
	1.6 Exercises	22
	1.7 Notes	23
2	On the fractional parts of powers of real numbers	24
	2.1 Thue, Hardy, Pisot, and Vijayaraghavan	25
	2.2 On some exceptional pairs (ξ, α)	31
	2.3 On the powers of real numbers close to 1	37
	2.4 On the powers of some transcendental numbers	42
	2.5 A theorem of Furstenberg	46
	2.6 A conjecture of de Mathan and Teulié	49
	2.7 Exercises	51
	2.8 Notes	53
3	On the fractional parts of powers of algebraic numbers	58
	3.1 The integer case	59
	3.2 Mahler's Z -numbers	60
	3.3 On the fractional parts of powers of algebraic numbers	63
	3.4 On the fractional parts of powers of Pisot and Salem numbers	67

3.5	The sequence $(\ \xi\alpha^n\)_{n \geq 1}$	71
3.6	Constructions of Pollington and of Dubickas	78
3.7	Waring's problem	82
3.8	On the integer parts of powers of algebraic numbers	83
3.9	Exercises	84
3.10	Notes	85
4	Normal numbers	88
4.1	Equivalent definitions of normality	89
4.2	The Champernowne number	96
4.3	Normality and uniform distribution	99
4.4	Block complexity and richness	101
4.5	Rational approximation to Champernowne-type numbers	102
4.6	Exercises	106
4.7	Notes	106
5	Further explicit constructions of normal and non-normal numbers	112
5.1	Korobov's and Stoneham's normal numbers	112
5.2	Absolutely normal numbers	121
5.3	Absolutely non-normal numbers	122
5.4	On the random character of arithmetical constants	124
5.5	Exercises	125
5.6	Notes	126
6	Normality to different bases	128
6.1	Normality to a prescribed set of integer bases	128
6.2	Normality to non-integer bases	133
6.3	On the expansions of a real number to two different bases	141
6.4	On the representation of an integer in two different bases	145
6.5	Exercises	145
6.6	Notes	146
7	Diophantine approximation and digital properties	149
7.1	Exponents of Diophantine approximation	150
7.2	Prescribing simultaneously the values of all the exponents v_b	154
7.3	Badly approximable numbers to integer bases	157
7.4	Almost no element of the middle third Cantor set is very well approximable	163

7.5	Playing games on the middle third Cantor set	166
7.6	Elements of the Cantor set with prescribed irrationality exponent	168
7.7	Normal and non-normal numbers with prescribed Diophantine properties	171
7.8	Hausdorff dimension of sets with missing digits	173
7.9	Exercises	176
7.10	Notes	177
8	Digital expansion of algebraic numbers	181
8.1	A transcendence criterion	182
8.2	Block complexity of algebraic numbers	184
8.3	Zeros in the b -ary expansion of algebraic numbers	187
8.4	Number of digit changes in the b -ary expansion of algebraic numbers	192
8.5	On the b -ary expansion of e and some other transcendental numbers	195
8.6	On the digits of the multiples of an irrational number	196
8.7	Exercises	200
8.8	Notes	202
9	Continued fraction expansions and β-expansions	206
9.1	Normal continued fractions	206
9.2	On the continued fraction expansion of an algebraic number	212
9.3	On β -expansions	217
9.4	Exercises	221
9.5	Notes	221
10	Conjectures and open questions	226
	<i>Appendix A</i> Combinatorics on words	235
	<i>Appendix B</i> Some elementary lemmata	243
	<i>Appendix C</i> Measure theory	246
	<i>Appendix D</i> Continued fractions	253
	<i>Appendix E</i> Diophantine approximation	258
	<i>Appendix F</i> Recurrence sequences	265
	References	269
	Index	310