Cluster-Tilting Theory

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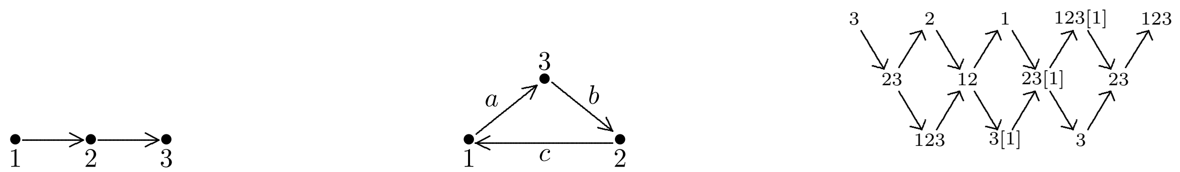
Tilting theory provides a good method for comparing two categories, such as module categories of finite-dimensional algebras. For an introduction, see e.g. [A]. BGP reflection functors [BGP] give a way of comparing the representation categories of two quivers, where one is obtained from the other by reversing all of the arrows incident with a sink or source. Auslander, Platzeck and Reiten [APR] showed that the BGP reflection functors can be realised directly as functors of the form $\mathrm{Hom}(T, -)$, where T is an APR-tilting module.

In cluster–tilting theory [BMR1, BMR2, BMRRT] APR-tilting theory has been generalised to cover arbitrary vertices of the quiver. This is done via the definition of a new category, known as the cluster category, as a quotient of the bounded derived category the module category of the path algebra of a quiver. See also [CC, CCS1, CCS2]; in [CCS1] a nice geometric combinatorial description of the cluster category in type A is given.

The cluster category is a triangulated category by a result of Keller [K]. The "cluster-tilted" algebras arising from this approach form a new class of algebras with strong connections to path algebras. By a result in [BMR2], the mutation rule for cluster algebras [FZ1] describes the change in the quiver of the algebra when one algebra is cluster-tilted to another. For an example of cluster-tilting, see Figure 1 (in Figure 1(b), the relations are that the product of any two arrows is zero).

My talks will be an introduction to triangulated categories and the necessary theory for the derived categories mentioned above, in order to define the cluster category and describe its main properties. I will also discuss cluster-tilting theory and the connection with cluster algebras.



(a) A quiver Q of type A_3.

(b) The quiver Q' of the algebra kQ cluster-tilted at the vertex 2.

(c) The Auslander-Reiten quiver of the cluster category.

Figure 1: Cluster-Tilting in type A_3

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