UNIVERSITY OF DUBLIN

XMA1131

TRINITY COLLEGE

Faculty of Science

school of mathematics

JF Mathematics
JF Theoretical Physics
JF Two Subject Mod

Michaelmas Term 2008

Course 113

Wednesday, December 10 Regent House 9.30–11.00

Dr. Vladimir Dotsenko

For each task, the number of points you can get for a complete solution of that task is printed next to it.

You may use all statements proved in class and in home assignments; when using some statement, you should formulate it clearly, e.g. "in class, we proved that if A is invertible, then the reduced row echelon form of A is the identity matrix".

Log tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

1. (14 points) Is the vector $\mathbf{n} = (2, 4, 1)$ parallel to the intersection line of the plane α passing through the points $(0, 1, -1)$, $(5, 1, -3)$, and $(6, -3, 3)$ and the plane β passing through the points $(2, 1, -1)$, $(5, 1, -3)$, and $(1, -2, 3)$? Why?

2. In this question, A and B are $n \times n$-matrices. For each of the following statements, prove it if it is true, and give a counterexample if it is false.

   (a) (10 points) $A^2 - B^2 = (A + B)(A - B)$ if and only if $AB = BA$.

   (b) (12 points) If $A^2 = B^2$, then $A = B$ or $A = -B$.

3. Consider the system of linear equations

$$\begin{cases} 2x_1 + x_2 - x_3 = 4, \\ x_1 + x_3 = 3, \\ x_1 - 3x_2 + 5x_3 = 1. \end{cases}$$

   (a) (10 points) Write down its matrix A, and its extended matrix A_+. Compute $\det(A)$, and show that A is invertible.

   (b) (10 points) Bring the matrix A_+ to its reduced row echelon form, and use the latter to solve the system.

   (c) (10 points) Show how to solve the system using the Cramer's rule.

4. (14 points) Determine all values of i, j, k and l for which the product

$$a_{i1}a_{24}a_{k5}a_{73}a_{42}a_{6j}a_{1l}$$

   occurs in the expansion of the 7×7-determinant with coefficient -1.

5. (20 points) Determine all values of x for which the matrix

$$\begin{pmatrix} 2-x & 1 & 0 \\ -1 & -x & 1 \\ 5 & 5 & 3-x \end{pmatrix}$$

   is not invertible.