UNIVERSITY OF DUBLIN

XMA1131

TRINITY COLLEGE

Faculty of Science

school of mathematics

JF Mathematics
JF Theoretical Physics
JF Two Subject Mod

Michaelmas Term 2008

Course 113, a sample exam paper

Dr. Vladimir Dotsenko

For each task, the number of points you can get for a complete solution of that task is printed next to it.

You may use all statements proved in class and in home assignments; when using some statement, you should formulate it clearly, e.g. "in class, we proved that if A is invertible, then the reduced row echelon form of A is the identity matrix".

Log tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

1. (14 points) Consider the plane α defined by the equation $2x + 3y + z = -3$ and the plane β passing through the points $(1, -2, 1)$, $(4, 0, 2)$, and $(2, -2, 1)$. Find the angle between the intersection line of α and β and the vector $\mathbf{a} = (1, 0, 1)$.

2. Consider the system of linear equations

$$\begin{cases} x_1 + 4x_2 + 2x_3 = 1, \\ x_1 + x_2 - x_3 = 3, \\ 5x_1 - x_2 + x_3 = 3. \end{cases}$$

   (a) (11 points) Write down its matrix A. Compute $\det(A)$, and show that A is invertible.

   (b) (11 points) Show how to compute the inverse matrix A^{-1} using row operations, and how to use the inverse matrix to solve the system.

   (c) (11 points) Show how to compute the inverse matrix A^{-1} using the adjoint matrix.

3. (10 points) Is the permutation $\begin{pmatrix} 1 & 4 & 3 & 5 & 10 & 8 & 9 & 6 & 7 & 2 \\ 3 & 4 & 5 & 10 & 9 & 8 & 7 & 6 & 2 & 1 \end{pmatrix}$ even or odd? Why?

4. (18 points) Prove that for an $n \times n$-matrix A, the system $Ax = 0$ has a nontrivial solution if and only if the system $A^4 x = 0$ has a nontrivial solution.

5. (25 points)

$$A_n = \begin{pmatrix} 2 & -1 & 0 & 0 & \dots & 0 \\ -1 & 2 & -1 & 0 & \dots & 0 \\ 0 & -1 & 2 & -1 & \dots & 0 \\ \vdots & \vdots & \dots & \ddots & \dots & \vdots \\ 0 & 0 & \dots & -1 & 2 & -1 \\ 0 & 0 & \dots & \dots & -1 & 2 \end{pmatrix}$$

   is the $n \times n$-matrix for which all elements on the main diagonal are equal to 2, all elements on the diagonals next to the main are equal to -1, all other elements are equal to 0. Show that $\det(A_n) = 2\det(A_{n-1}) - \det(A_{n-2})$ for all $n \geq 3$, and use this formula to compute $\det(A_n)$.