MA 1212: Linear Algebra II
Tutorial problems, January 22, 2015

1. (**a**) A subspace U of $\mathbb{R}^4$ is the linear span of the vectors $\begin{pmatrix} 1 \\ 0 \\ -3 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 4 \\ 1 \\ 1 \end{pmatrix}$, and $\begin{pmatrix} 0 \\ 2 \\ -5 \\ -1 \end{pmatrix}$. Find some basis of this subspace.

(**b**) A subspace W of $\mathbb{R}^4$ is the linear span of the vectors $\begin{pmatrix} -2 \\ 2 \\ 1 \\ 3 \end{pmatrix}$, $\begin{pmatrix} -1 \\ -2 \\ -3 \\ -4 \end{pmatrix}$, $\begin{pmatrix} 0 \\ -3 \\ -1 \\ 1 \end{pmatrix}$, and $\begin{pmatrix} -3 \\ -6 \\ -4 \\ 1 \end{pmatrix}$. Find some basis of this subspace.

2. Find some basis for the intersection $U \cap W$ of the subspaces from the previous question.

3. Is the subspace spanned by the vectors $v_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ an invariant subspace of the linear transformation φ of $\mathbb{R}^3$ given by the matrix $A = \begin{pmatrix} -4 & 4 & 5 \\ 16 & 2 & -6 \\ -16 & 1 & 9 \end{pmatrix}$? Explain your answer.