

MA 1212: Linear Algebra II
Tutorial problems, March 26, 2015

1. The sequence $\mathbf{a}_1, \mathbf{a}_2, \dots$ is defined recursively: $\mathbf{a}_{n+2} = 8\mathbf{a}_{n+1} - 16\mathbf{a}_n$, $\mathbf{a}_0 = 1$, $\mathbf{a}_1 = 1$. Find an explicit formula for \mathbf{a}_n .

2. Do the following two matrices represent the same linear transformations relative to different bases? Explain your answer. (*Hint*: two matrices represent the same linear transformations relative to different bases if their Jordan normal forms are the same; note that you only need to determine the Jordan normal form (sizes of blocks for various eigenvalues), and not a Jordan basis).

(a) $A = \begin{pmatrix} 0 & 7 & 1 \\ -1 & 4 & 1 \\ 0 & 3 & 1 \end{pmatrix};$

(b) $B = \begin{pmatrix} -3 & 5 & 5 \\ -1 & 3 & 1 \\ -3 & 3 & 5 \end{pmatrix}.$

3. Assume that for a $\mathbf{n} \times \mathbf{n}$ -matrix A with real matrix elements we have $A^2 = -I$. Prove that $\text{tr } A = 0$.