MA 1111: Linear Algebra I
Selected answers/solutions to the assignment due November 5, 2015

1. (**a**) For $x = 0$, the first column is proportional to the third one. For $x = 2$, the first column is proportional to the second one.

(**b**) The third row is equal to the sum of the first two, so the determinant is equal to zero.

2. (**a**) $M_{11} = 12$, $M_{12} = -4$, $M_{13} = -8$, $M_{21} = 19$, $M_{22} = 7$, $M_{23} = -10$, $M_{31} = 9$, $M_{32} = 5$, $M_{33} = 2$.

(**b**) $C_{11} = 12$, $C_{12} = 4$, $C_{13} = -8$, $C_{21} = -19$, $C_{22} = 7$, $C_{23} = 10$, $C_{31} = 9$, $C_{32} = -5$, $C_{33} = 2$.

3. (**a**) $2 \cdot 12 + 4 \cdot 4 - 8 = 32$; (**b**) $2 \cdot 12 - 19 + 3 \cdot 9 = 32$; (**c**) $-19 + 3 \cdot 7 + 3 \cdot 10 = 32$; (**d**) $4 \cdot 4 + 3 \cdot 7 - 5 = 32$; (**e**) $3 \cdot 9 - 5 + 5 \cdot 2 = 32$; (**f**) $1 \cdot (-8) + 3 \cdot 10 + 5 \cdot 2 = 32$; (**g**) The adjugate matrix is equal to $\begin{pmatrix} 12 & -19 & 9 \\ 4 & 7 & -5 \\ -8 & 10 & 2 \end{pmatrix}$, and the inverse matrix is $\frac{1}{32}\begin{pmatrix} 12 & -19 & 9 \\ 4 & 7 & -5 \\ -8 & 10 & 2 \end{pmatrix}$. (**h**) We have $\det\begin{pmatrix} 1 & 4 & 1 \\ -1 & 3 & 3 \\ 1 & 1 & 5 \end{pmatrix} = 40$, $\det\begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 3 \\ 3 & 1 & 5 \end{pmatrix} = -8$, and $\det\begin{pmatrix} 2 & 4 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & 1 \end{pmatrix} = -16$, so by Cramer's rule the only solution to this system is $\begin{pmatrix} 5/4 \\ -1/4 \\ -1/2 \end{pmatrix}$.

4. (**a**) We have $1 = \det(I_n) = \det(A^{-1}A) = \det(A^{-1})\det(A)$, so $\det(A^{-1}) = \frac{1}{\det(A)}$.

(**b**) We have $\det(A^{-1}BA) = \det(A^{-1})\det(B)\det(A) = \det(A)^{-1}\det(B)\det(A) = \det(B)$.

5. (**a**) We have $(-1)^n = \det(-I_n) = \det(A^2) = \det(A)^2$. Since $\det(A)$ is a real number, $\det(A)^2 \geqslant 0$. Hence, n is even to ensure $(-1)^n \geqslant 0$.

(**b**) We have $\det(A^T) = \det(A)$ and at the same time $\det(A^T) = \det(-A) = (-1)^{2k+1}\det(A)$ (since we can take the factor -1 from each row), therefore $\det(A) = -\det(A)$, so $\det(A) = 0$.