MA 1111: Linear Algebra I
Homework problems due October 29, 2015

Solutions to this problem sheet are to be handed in after our class at 3pm on Thursday. Please attach a cover sheet with a declaration `http://tcd-ie.libguides.com/plagiarism/declaration` confirming that you know and understand College rules on plagiarism.

Please put your name and student number on each of the sheets you are handing in. (Or, ideally, staple them together).

1. Find all i, j, k, l for which the permutation $\begin{pmatrix} 5 & 2 & k & 3 & l & 1 \\ 4 & 1 & 3 & i & 6 & j \end{pmatrix}$ is even.

2. (**a**) Using the property $\operatorname{tr}(AB) = \operatorname{tr}(BA)$ (see the previous homework), prove that if A and B are two $n \times n$-matrices, and A is invertible, then $\operatorname{tr}(A^{-1}BA) = \operatorname{tr}(B)$.

(**b**) Use the previous question to show that if for two $n \times n$-matrices A and B we have $AB - BA = A$, then A is not invertible.

3. Compute the determinant of the matrix (**a**) $\begin{pmatrix} 1 & 0 & -2 \\ 1 & 1 & 3 \\ 4 & 3 & 1 \end{pmatrix}$; (**b**) $\begin{pmatrix} 1 & 1 & -2 & -1 \\ 2 & 0 & 3 & -1 \\ 4 & 2 & 3 & 1 \\ 3 & 0 & 0 & 1 \end{pmatrix}$.

4. For which values of c does A fail to be invertible:
(**a**) $A = \begin{pmatrix} 2-c & -1 \\ -1 & 2-c \end{pmatrix}$; (**b**) $A = \begin{pmatrix} 2 & 1 & 3 \\ c-1 & 2+c & 4c \\ 1 & 3 & -1 \end{pmatrix}$.

5. Which of the following statements are true (for $n \times n$-matrices):

1. If AB is invertible, then A is invertible and B is invertible;
2. If A is invertible and B is invertible, then AB is invertible;
3. If $AB = 0$ then either $A = 0$ or $B = 0$;
4. If $AB = 0$ then neither A nor B is invertible;
5. If $AB = 0$ then at least one of A and B is not invertible.

(In these statements, 0 means the matrix whose all entries are equal to zero). For those of these statements which are true, prove them, for the rest give counterexamples.