MA 1111: Linear Algebra I
Homework problems due November 19, 2015

Solutions to this problem sheet are to be handed in after our class at 3pm on Thursday. Please attach a cover sheet with a declaration `http://tcd-ie.libguides.com/plagiarism/declaration` confirming that you know and understand College rules on plagiarism.

Please put your name and student number on each of the sheets you are handing in. (Or, ideally, staple them together).

In problems 1–4, determine whether, for the given n, the vectors $\{v_i\}$ in $\mathbb{R}^n$ (i) are linearly independent; (ii) span $\mathbb{R}^n$; (iii) form a basis of $\mathbb{R}^n$.

1. (**a**) $n = 2$, $v_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $v_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

(**b**) $n = 2$, $v_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $v_3 = \begin{pmatrix} 7 \\ 6 \end{pmatrix}$.

2. $n = 3$, $v_1 = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$, $v_3 = \begin{pmatrix} 7 \\ 6 \\ -1 \end{pmatrix}$.

3. $n = 3$, $v_1 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$, $v_3 = \begin{pmatrix} 7 \\ 6 \\ 1 \end{pmatrix}$.

4. $n \geqslant 1$ arbitrary, $v_1 = \begin{pmatrix} 1 \\ 2 \\ \vdots \\ n \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 2^2 \\ 3^2 \\ \vdots \\ n^2 \end{pmatrix}$, $\ldots$, $v_n = \begin{pmatrix} 1 \\ 2^n \\ 3^n \\ \vdots \\ n^n \end{pmatrix}$.

5. Prove that if three vectors u, v, w in $\mathbb{R}^n$ are linearly independent, then the vectors $u-2w$, $v+w$, w are linearly independent as well.