MA 1111: Linear Algebra I
Homework problems due November 26, 2015

Solutions to this problem sheet are to be handed in after our class at 3pm on Thursday. Please attach a cover sheet with a declaration `http://tcd-ie.libguides.com/plagiarism/declaration` confirming that you know and understand College rules on plagiarism.

Please put your name and student number on each of the sheets you are handing in. (Or, ideally, staple them together).

1. Prove that if the three vectors u, v, w in $\mathbb{R}^n$ span $\mathbb{R}^n$, then the vectors $u-2w$, $v+w$, w span $\mathbb{R}^n$ as well. (*Hint*: it helps to represent the original vectors u, v, and w as linear combinations of these new vectors).

2. Which of the following subsets U of $\mathbb{R}^3$ are subspaces? Explain your answers.

(**a**) $U = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : 3x - 5y + z = 0 \right\}$.

(**b**) $U = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : 2x - y + z = 1 \right\}$.

(**c**) $U = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x^2 - y^2 + z^2 = 0 \right\}$.

3. For the system

$$\begin{cases} 2x_1 - 4x_2 + x_3 + x_4 = 0, \\ x_1 - 2x_2 + 5x_4 = 0, \end{cases}$$

find some vectors $v_1, \ldots, v_k$ such that the solution set to this system equals $\operatorname{span}(v_1, \ldots, v_k)$.

4. For the matrix $A = \begin{pmatrix} 1 & -2 & -1 & 1 \\ 2 & -1 & -2 & 1 \\ 0 & 6 & 0 & -2 \end{pmatrix}$, find some vectors $v_1, \ldots, v_k$ such that the solution set to the system $Ax = 0$ equals $\operatorname{span}(v_1, \ldots, v_k)$.

5. (**a**) Show that for every vector space V and every element $\mathbf{v} \in V$ the opposite element is unique: if $\mathbf{v} + \mathbf{x} = \mathbf{x} + \mathbf{v} = 0$ and $\mathbf{v} + \mathbf{y} = \mathbf{y} + \mathbf{v} = 0$, then $\mathbf{x} = \mathbf{y}$.

(**b**) Show that for every vector space V and every element $\mathbf{v} \in V$ we have $(-1) \cdot \mathbf{v} = -\mathbf{v}$.