MA 1111: Linear Algebra I
Homework problems due December 10, 2015

Solutions to this problem sheet are to be handed in after our class at 3pm on Thursday. Please attach a cover sheet with a declaration `http://tcd-ie.libguides.com/plagiarism/declaration` confirming that you know and understand College rules on plagiarism.

Please put your name and student number on each of the sheets you are handing in. (Or, ideally, staple them together).

By P_n we always denote the vector space of all polynomials in x of degree at most n.

1. (**a**) For the vector space $\mathbb{R}^3$, find the transition matrix $M_{\mathbf{e},\mathbf{f}}$ from the basis

$$e_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, e_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, e_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

to the basis

$$f_1 = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}, f_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, f_3 = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}.$$

(**b**) Given that a vector has coordinates $1, 7, -3$ with respect to the basis f_1, f_2, f_3, find its coordinates with respect to the basis e_1, e_2, e_3.

2. Write down the transition matrix between the bases $1, t, \ldots, t^n$ and $1, t+1, \ldots, (t+1)^n$ of P_n for $n = 1, 2, 3$.

3. Which of the following functions from P_3 to P_3 are linear maps? Explain your answers. (**a**) $f(t) \mapsto \frac{f(t)-f(0)}{t}$; (**b**) $f(t) \mapsto tf'(t) - 2f(t)$; (**c**) $f(t) \mapsto f''(t)f'(t) - t^2 f'''(t)$.

4. Let $v = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \in \mathbb{R}^3$.

(**a**) Show that the function from $\mathbb{R}^3$ to $\mathbb{R}^1$ given by

$$w \mapsto v \cdot w$$

is a linear map, and find the matrix of this linear map relative to the basis of standard unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ in $\mathbb{R}^3$ and the basis element (1) in $\mathbb{R}^1$.

(**b**) Show that the function from $\mathbb{R}^3$ to $\mathbb{R}^3$ given by

$$w \mapsto v \times w$$

is a linear transformation, and find the matrix of this linear transformation relative to the basis of standard unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$.

5. Given that the matrix of a linear transformation $\varphi\colon \mathbb{R}^2 \to \mathbb{R}^2$ relative to the basis $e_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $e_2 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ is equal to $\begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$, compute the matrix of φ relative to the basis $f_1 = \begin{pmatrix} 13 \\ -12 \end{pmatrix}$, $f_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.